RECENT DEVELOPMENTS IN NJL-JET MODEL: TMD

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CSSM

Collaborators:
A.W.Thomas, W. Bentz & I.Cloet

PacSPIN 2011 - Cairns: June 20-24, 2011

OUTLOOK

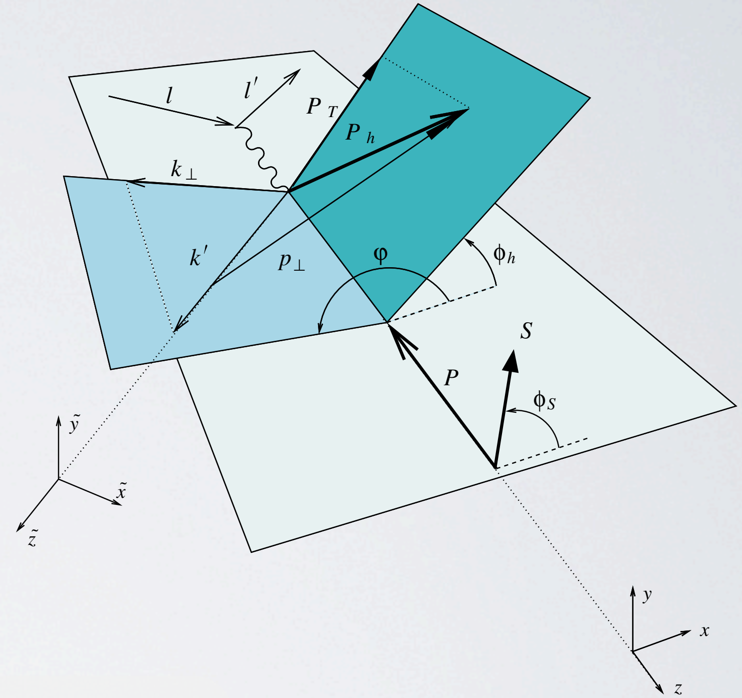
- Motivation
- Short Overview of the NJL-jet model:
  - Strange quark and Kaons
  - Monte-Carlo approach:
    - Vector mesons, Nucleon-Antinucleon channels, secondary hadrons from the decays of resonances.
- Transverse Momentum Dependent FF, Hadron TM in SIDIS.
- Dihadron Fragmentation Functions.
- Future Plans.

EXPLORING HADRON STRUCTURE

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): *e N* → *e h X*

- Cross-section factorizes into parton distribution and fragmentation functions.



Access to hadron structure:

- Ex., unpolarized cross section is ~

$$\sum_q e_q^2 \int d^2\mathbf{k}_\perp f_1^q(x, k_\perp) \pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp)$$

MOTIVATION

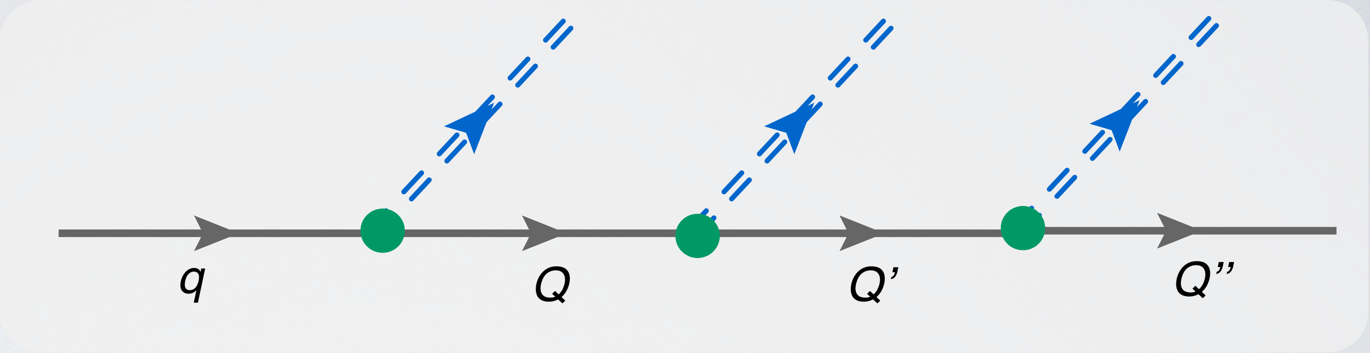
- Providing guidance based on a sophisticated model for applications to problems where phenomenology is difficult/ inadequate.
- Unfavored fragmentation functions from the model that goes beyond a single hadron emission approximation.
- Automatically satisfies the sum rules (at the model scale).
- Transverse-momentum dependent (TMD) fragmentations in the same model where structure functions (both unpolarized and polarized) were calculated.

THE QUARK JET MODEL

Field, Feynman.Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



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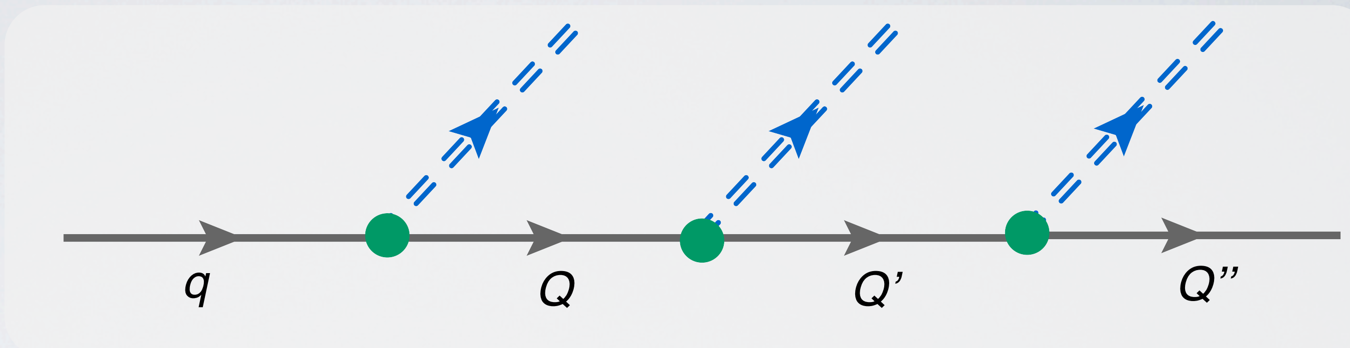


The probability of finding mesons m with mom. fraction z in a jet of quark q

$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m\left(\frac{z}{y}\right)\frac{dz}{y}$$

Probability of emitting the meson at link 1

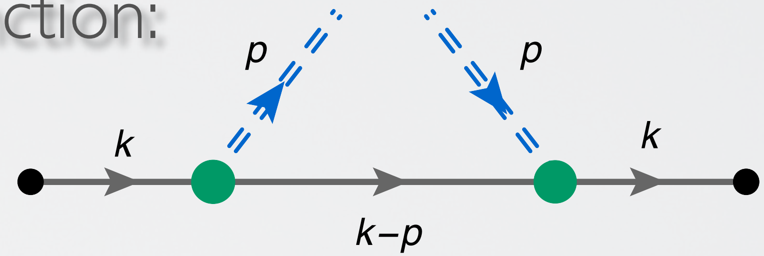
Probability of Momentum fraction y is transferred to jet at step 1

The probability scales with mom. fraction

NJL-JET: ELEMENTARY SPLITTING FUNCTIONS FROM NJL

- One-quark truncation of the wavefunction:

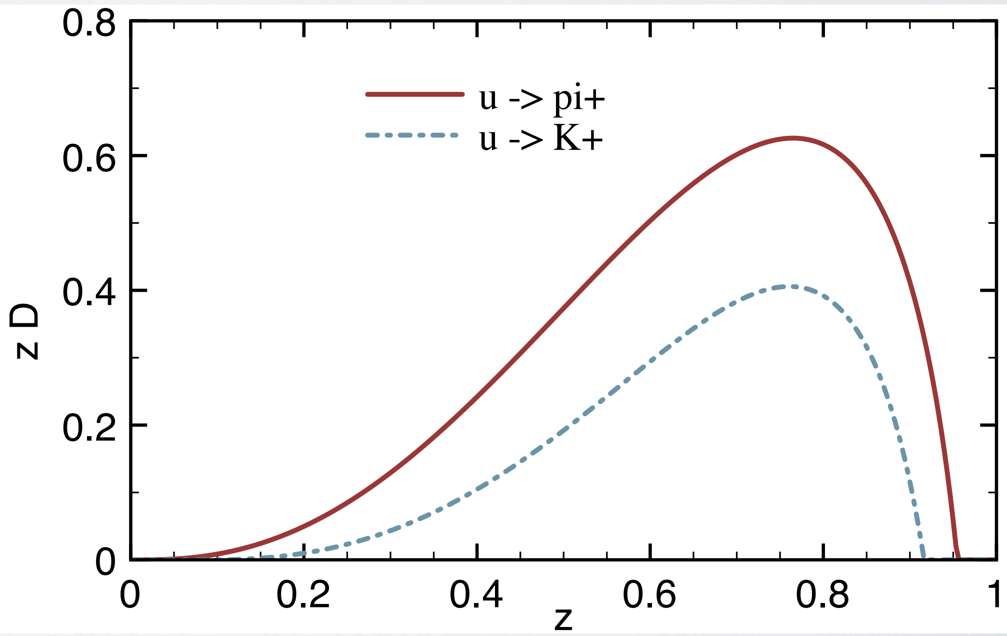
$$d_q^m(z) : q \to Qm \quad m = q\bar{Q}$$



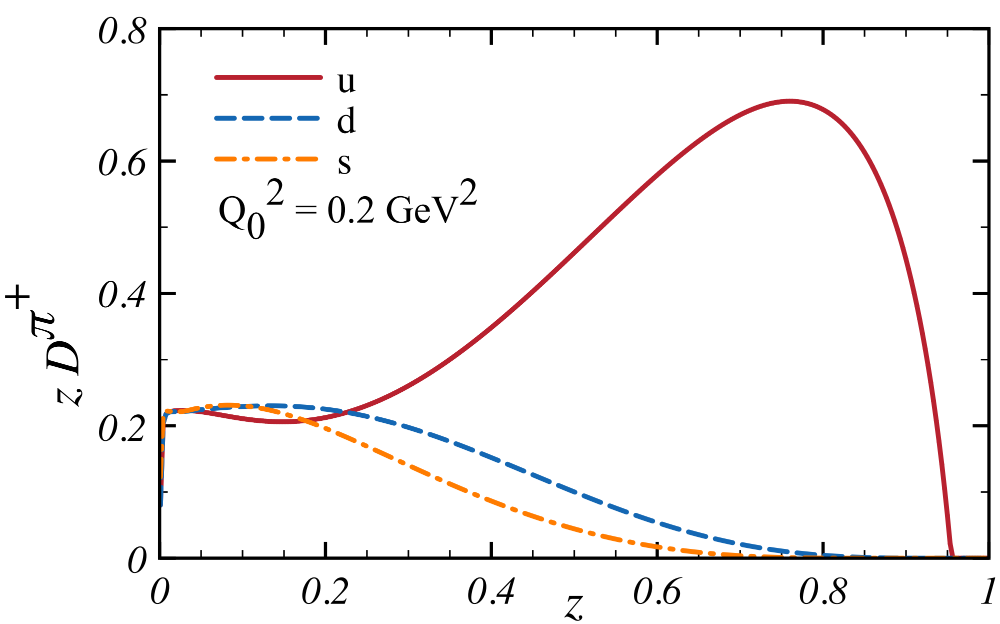
- Only 4-point interaction in the Lagrangian
- Lepage-Brodsky (LB)Invariant Mass Cutoff Regularization

$$u \to d\pi^+$$

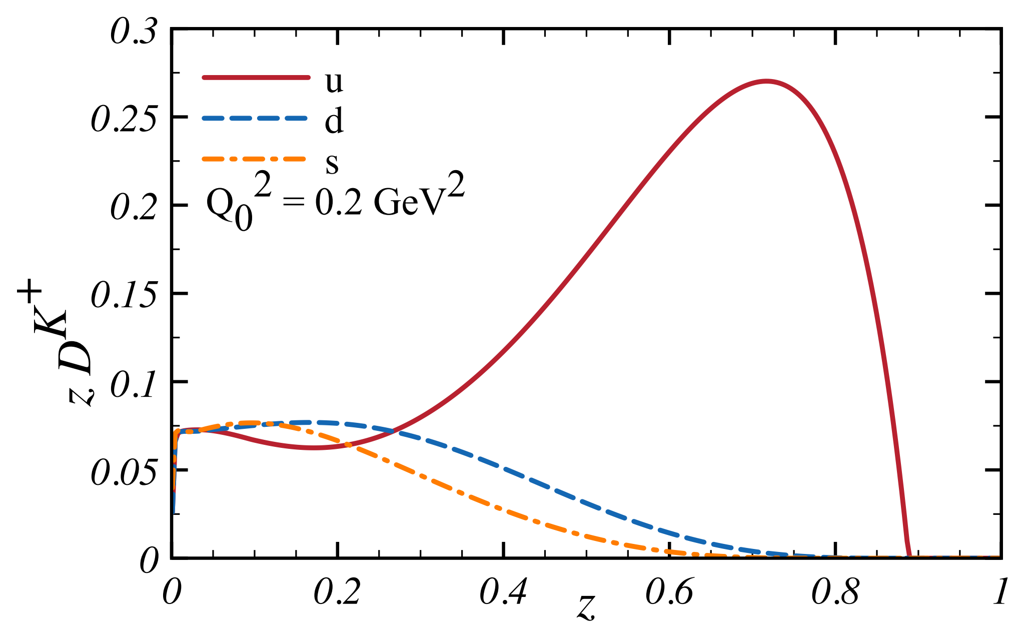
$$u \to sk^+$$



SOLUTIONS OF THE INTEGRAL EQUATIONS

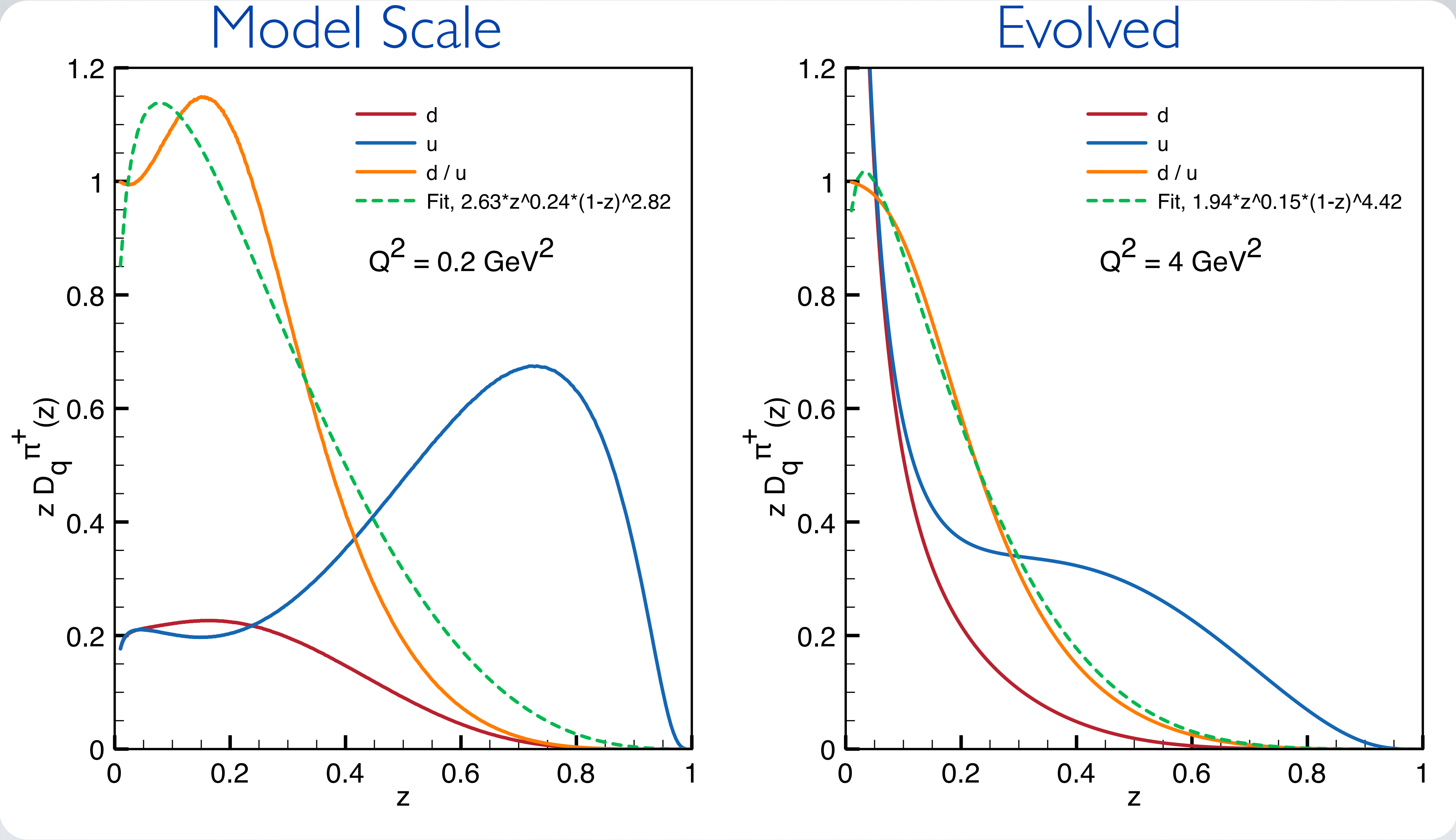


π^+



K^+

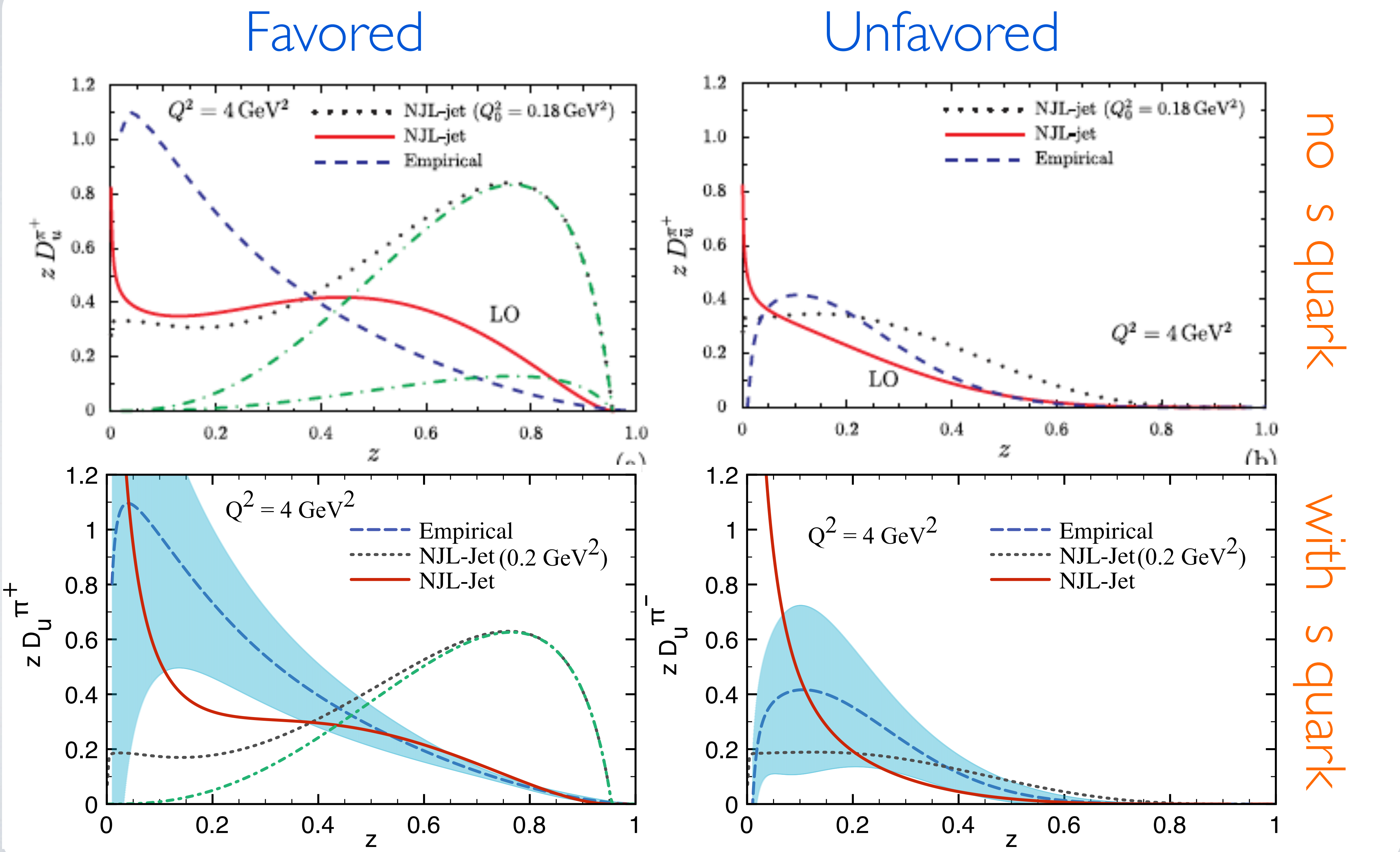
THE RATIO OF UNFAVORED TO FAVORED



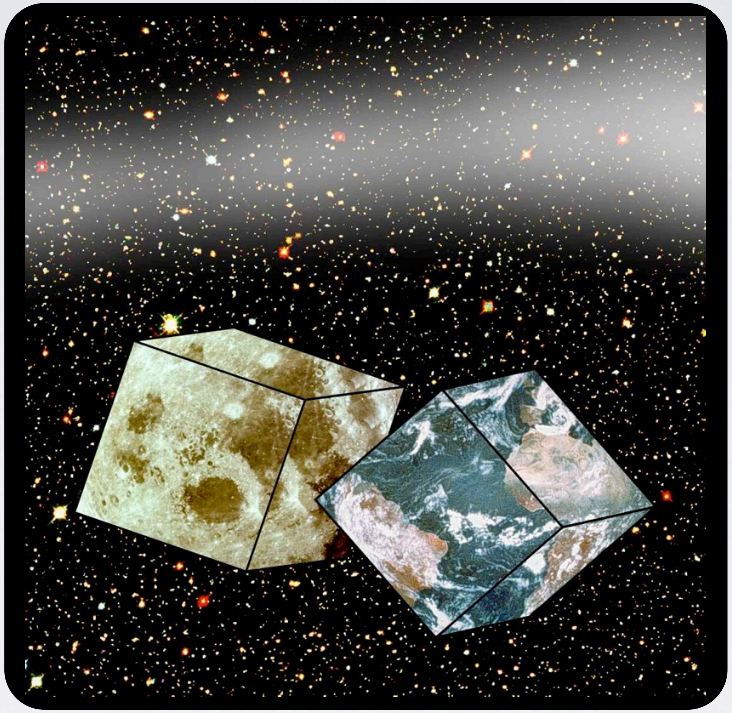
Fit Function - $f(z) = N z^{\alpha}(1-z)^{\beta}$

STRANGENESS EFFECT IN PION

Ito et al. Phys.Rev.D80:074008,2009



MONTE-CARLO (MC) APPROACH

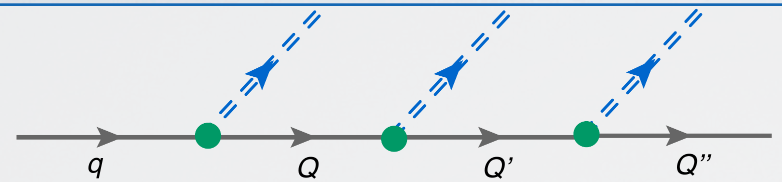


- Simulate decay chains to extract number densities.
- Allows for inclusion of TMD and experimental cut-offs.
- Numerically trivially parallelizeable (MPI, GPGPU).

FRAGMENTATIONS FROM MC

STARTING WITH PIONS

- Assume Cascade process:



$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

- Sample the emitted hadron according to splitting weight.
- Randomly sample z from input splittings.
- Evolve to sufficiently large number of decay links.
- Repeat for decay chains with the same initial quark.

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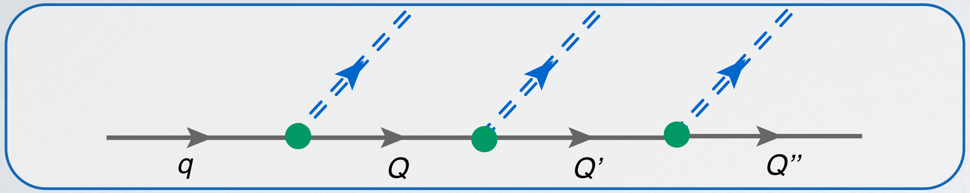

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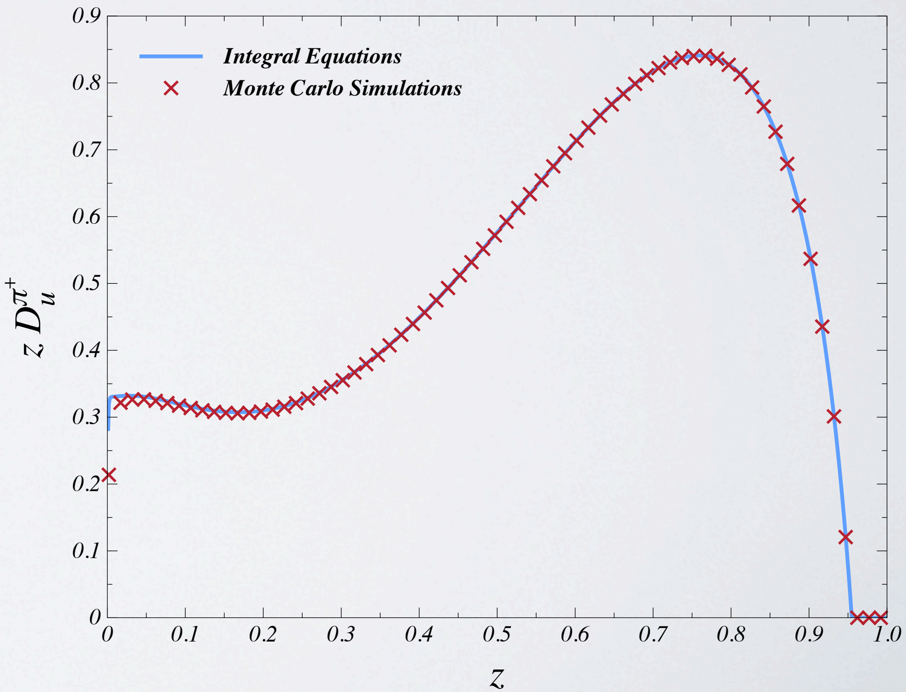
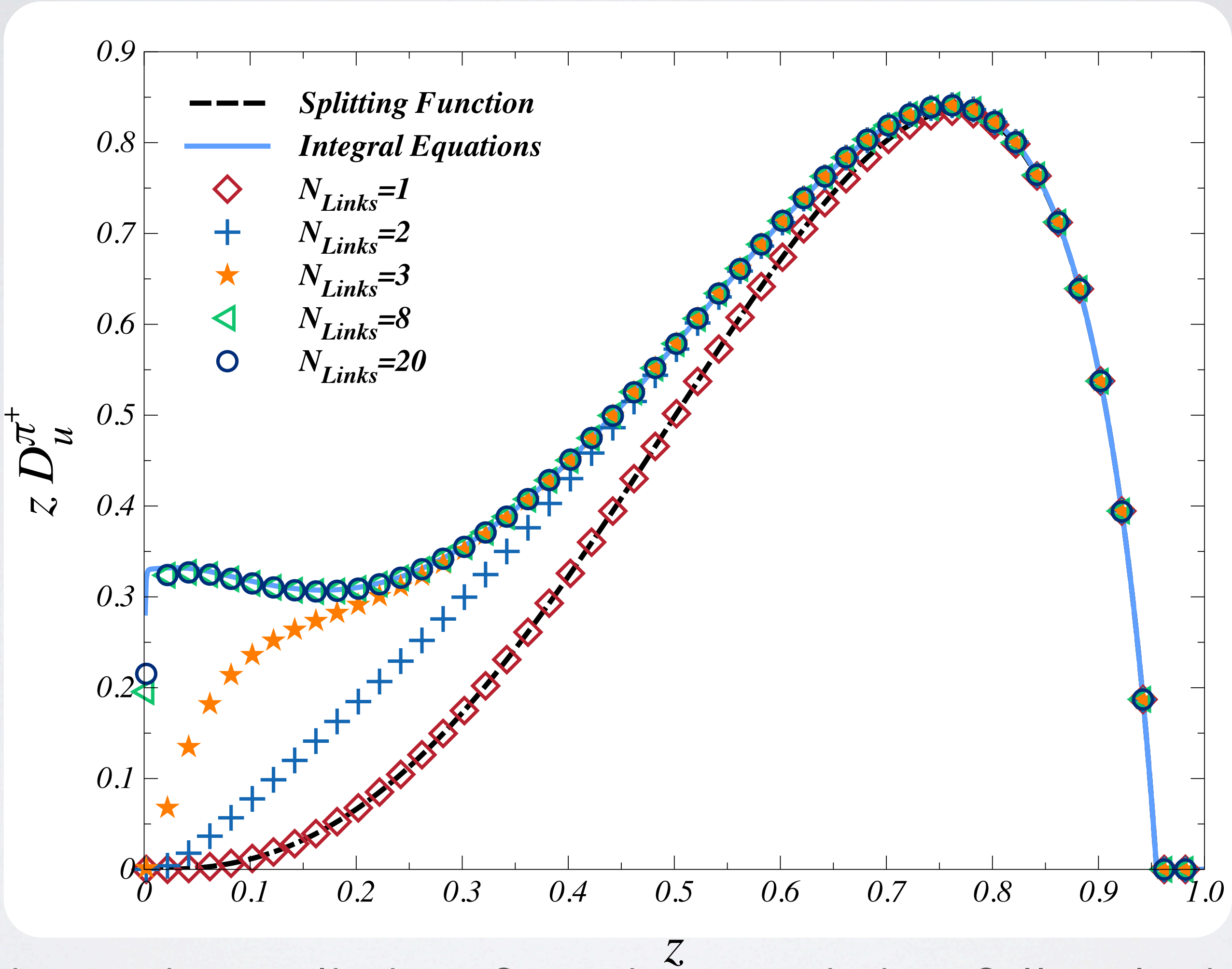


DEPENDENCE ON CHAIN CUTOFF

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.

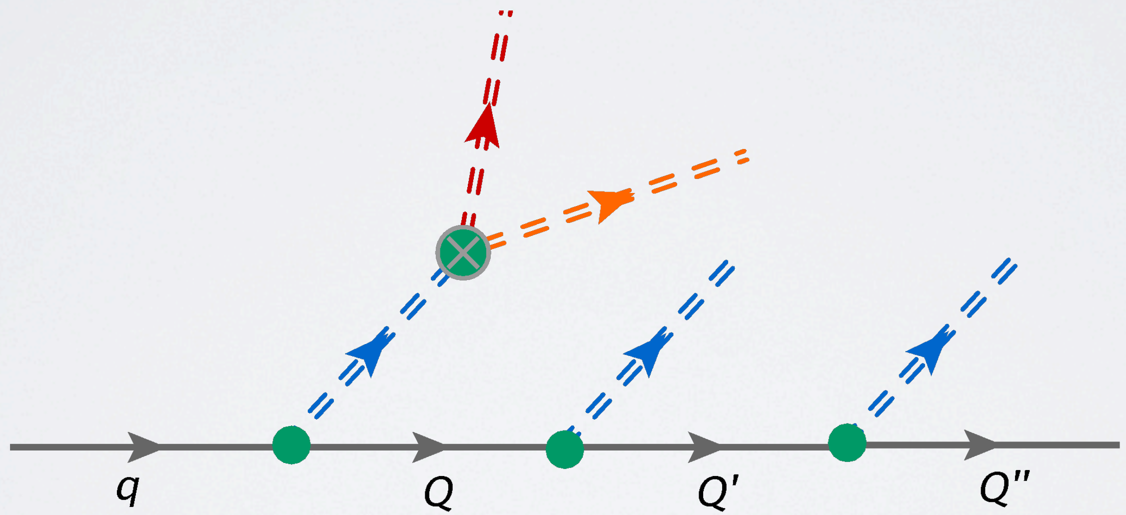
- The low z region is saturated with just a few emissions.

MORE CHANNELS: VECTOR MESONS

- Calculate quark splittings $d_q^m(z)$ in vector channel:

$$m = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi$$

- Add the decay of the resonances:

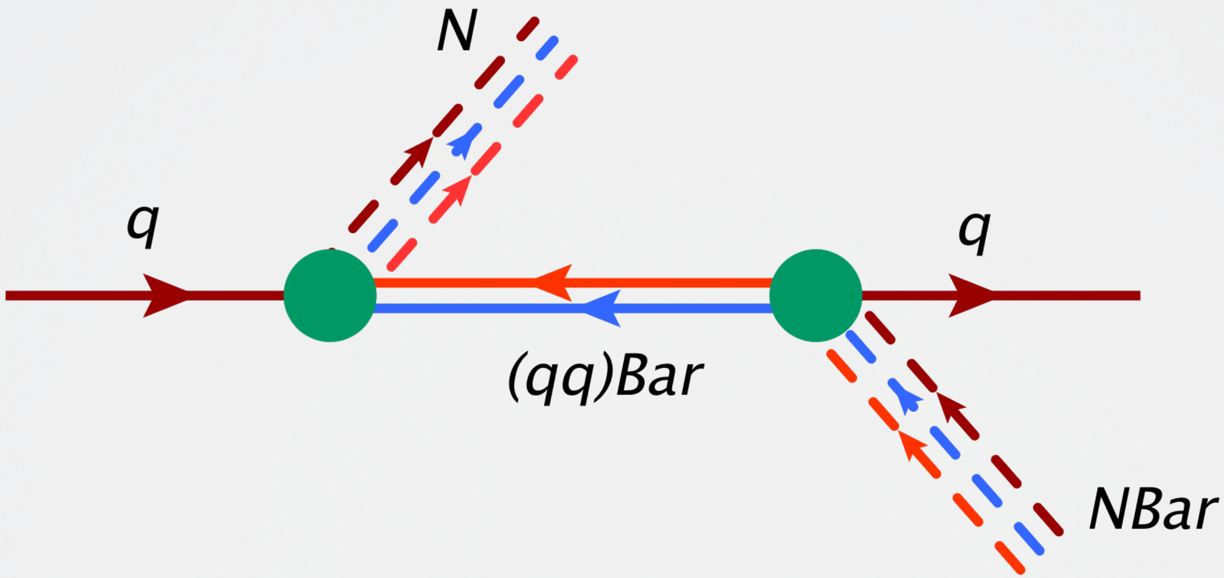


- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

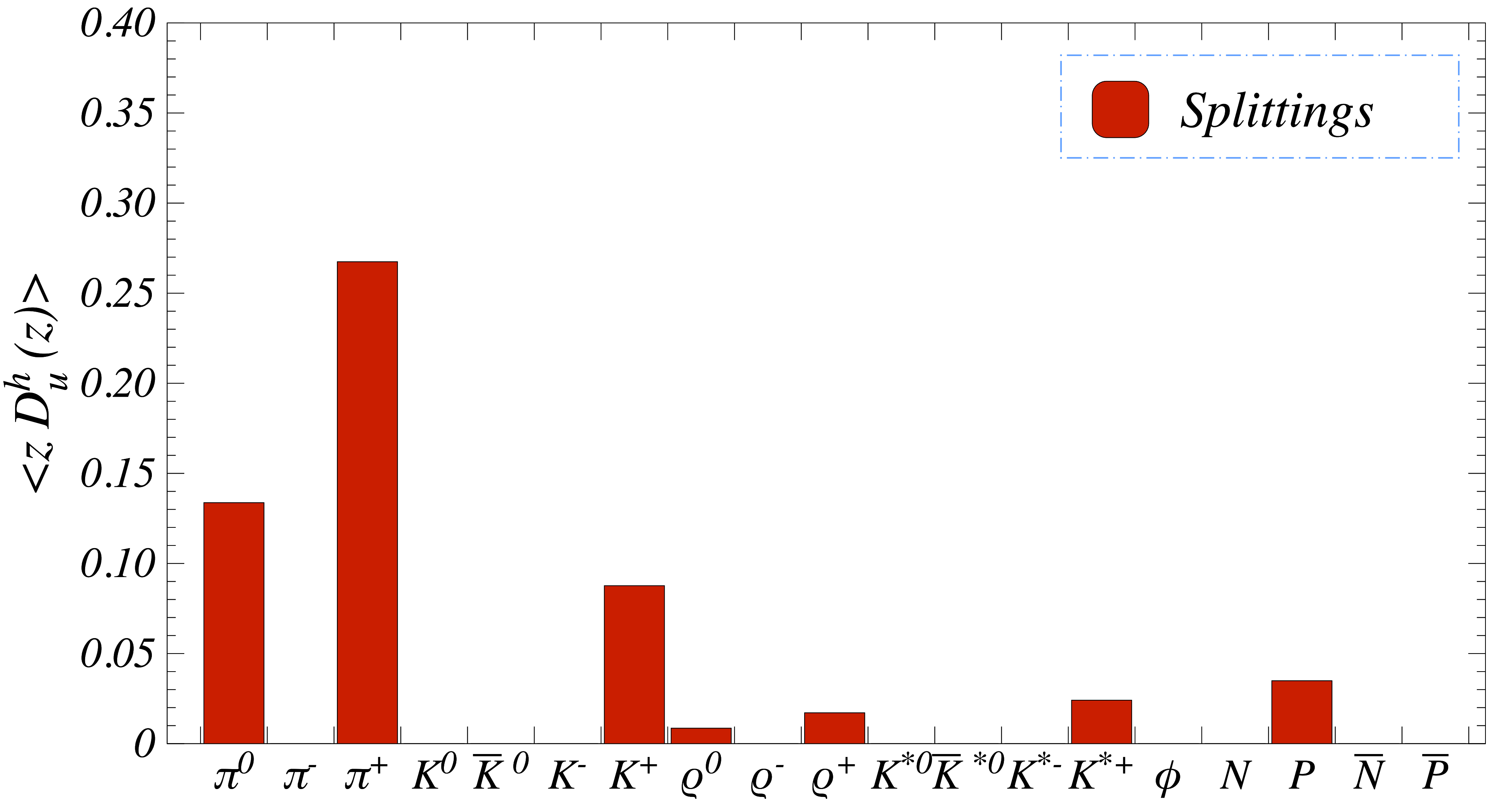
More Channels: Nucleon Anti-Nucleon

- Invoke quark-diquark model for nucleon.
- Calculate splittings $d_q^N(z)$ and $d_{\overline{qq}}^{\overline{N}}(z)$ (quark to nucleon and anti-diquark to anti-nucleon):



- We considered only scalar (anti-)diquarks (for now).

Results: Momentum Fractions



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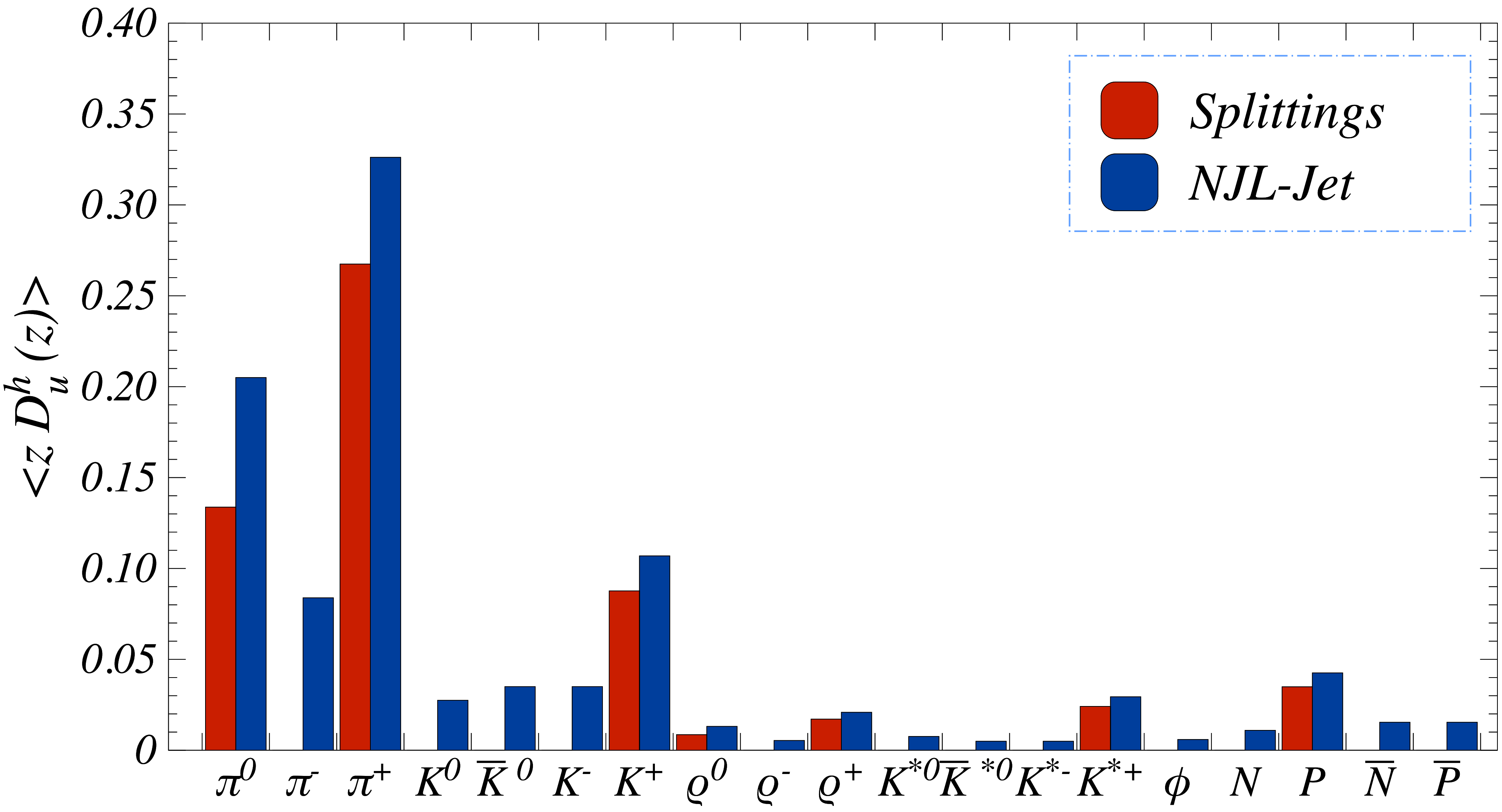

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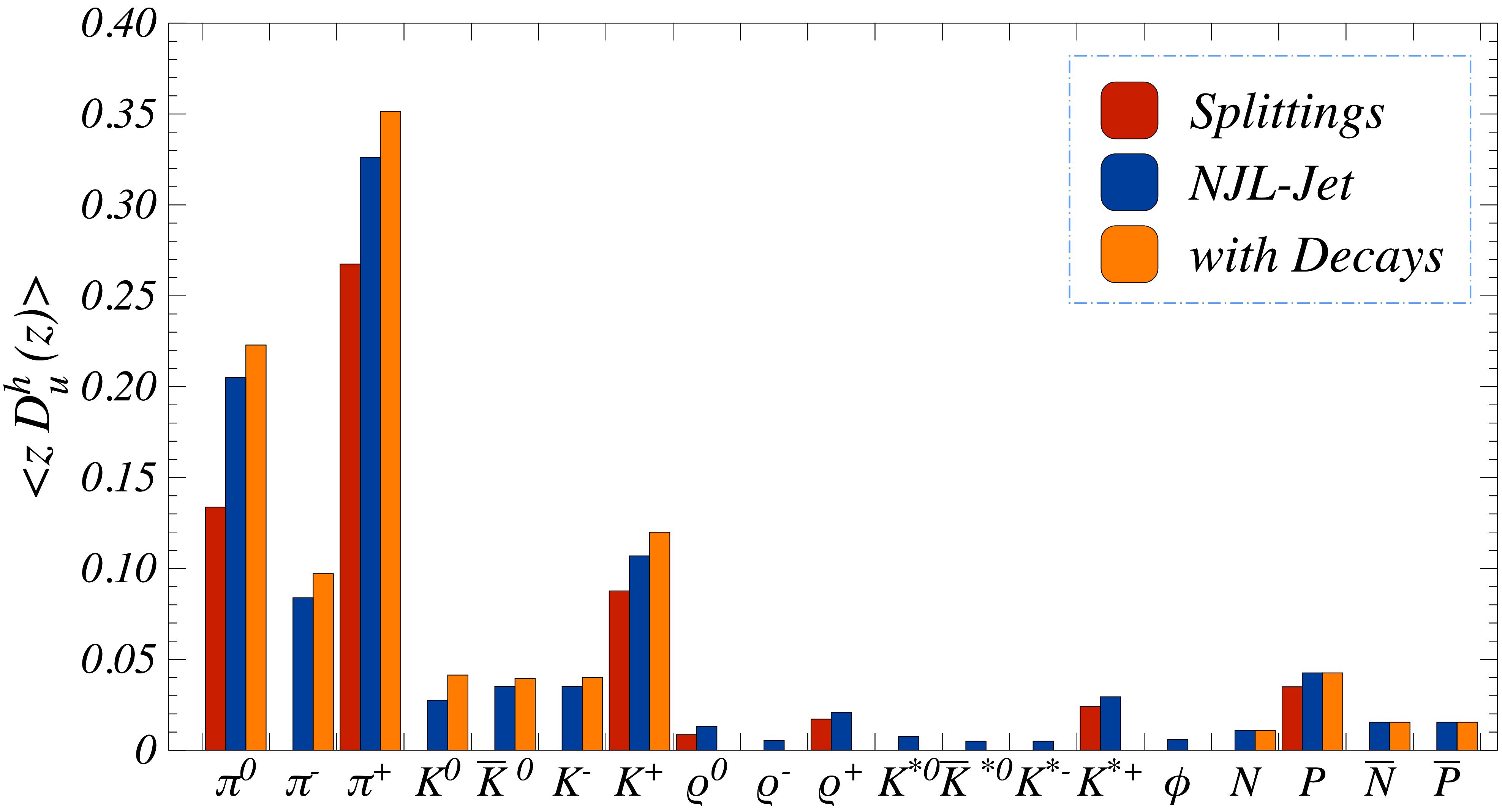

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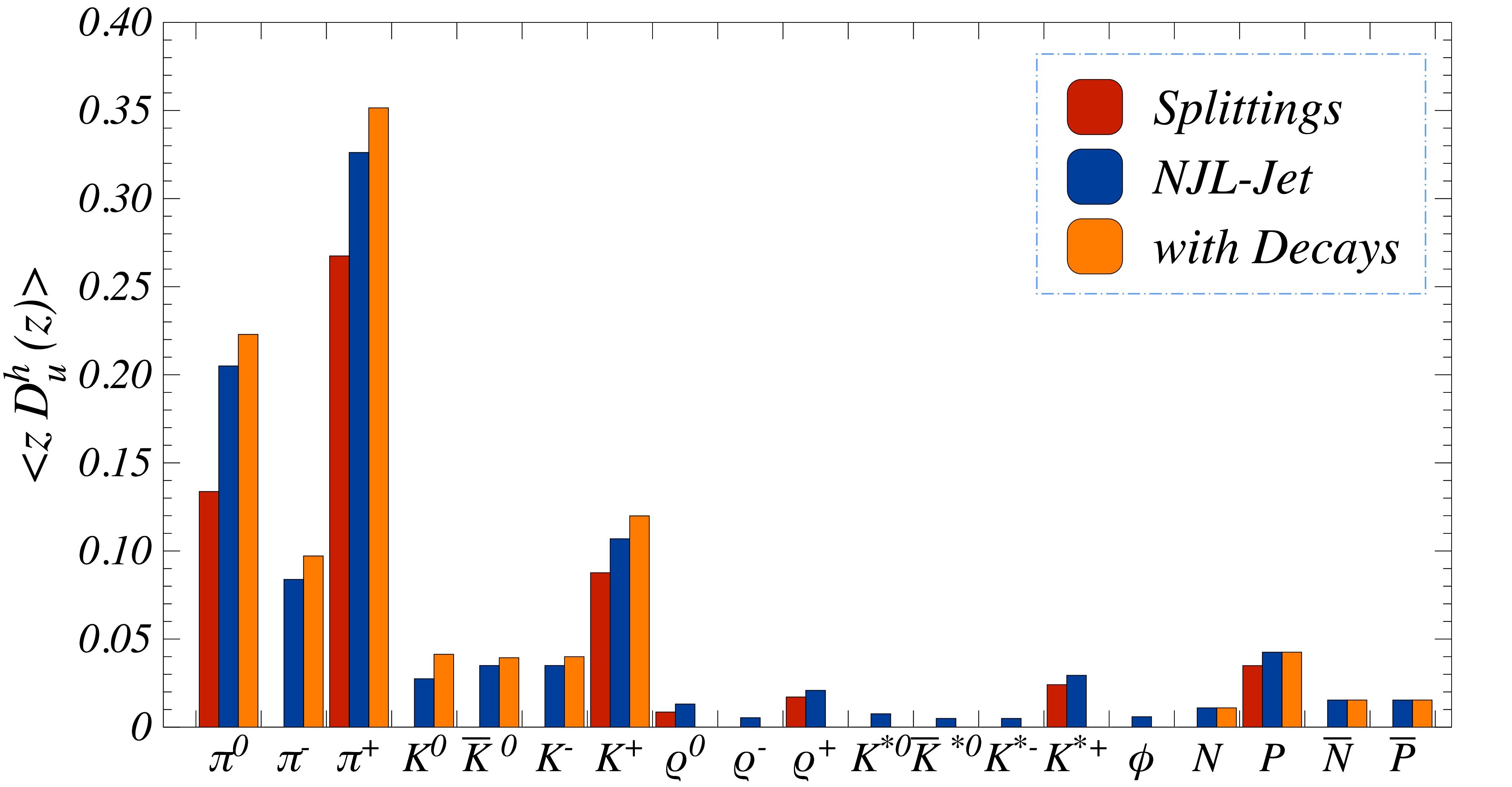
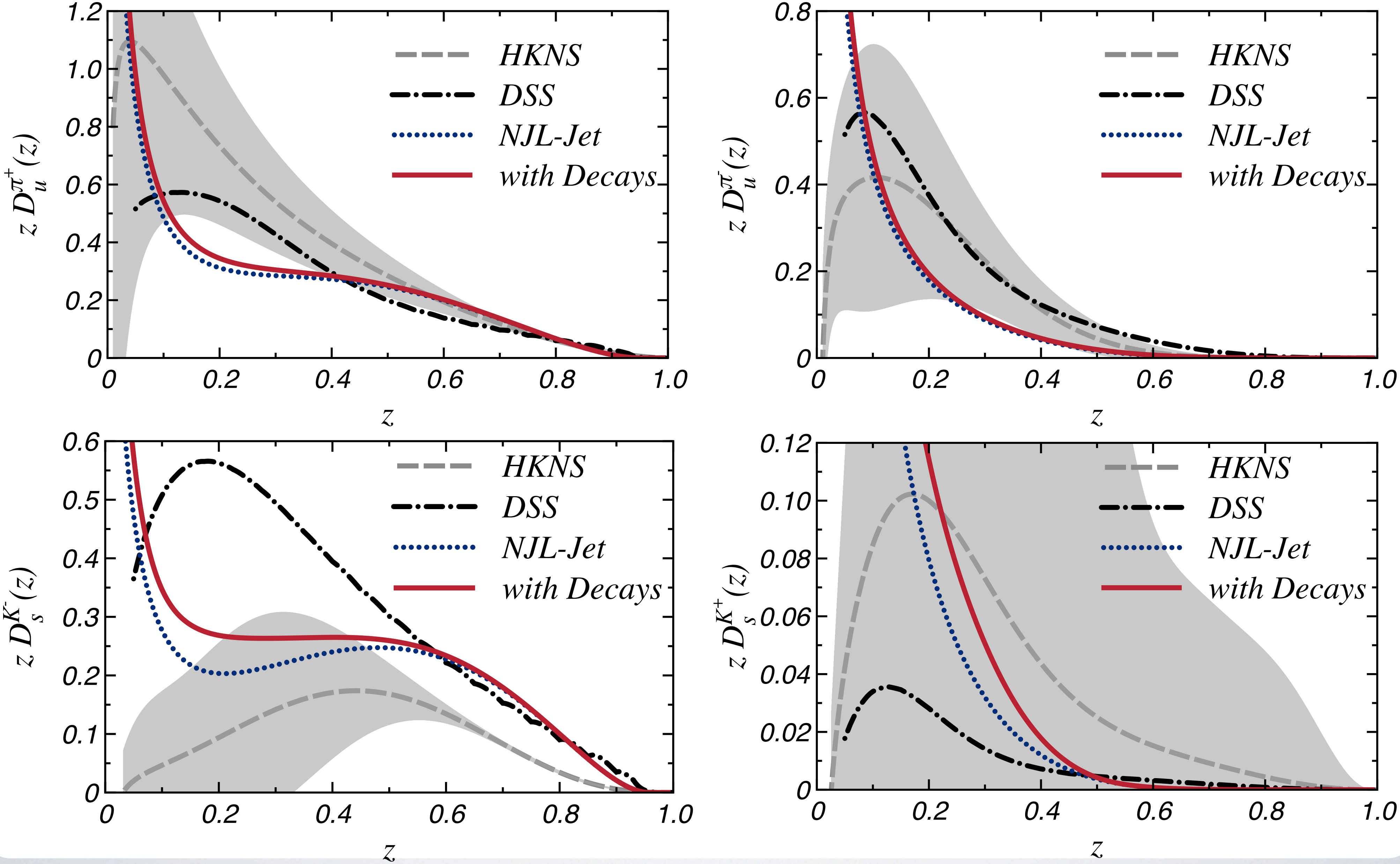


The Momentum (and Isospin) sumrules satisfied within numerical precision (less than 0.1 %)!

Results: $Q^2 = 4\ \mathrm{GeV}^2$

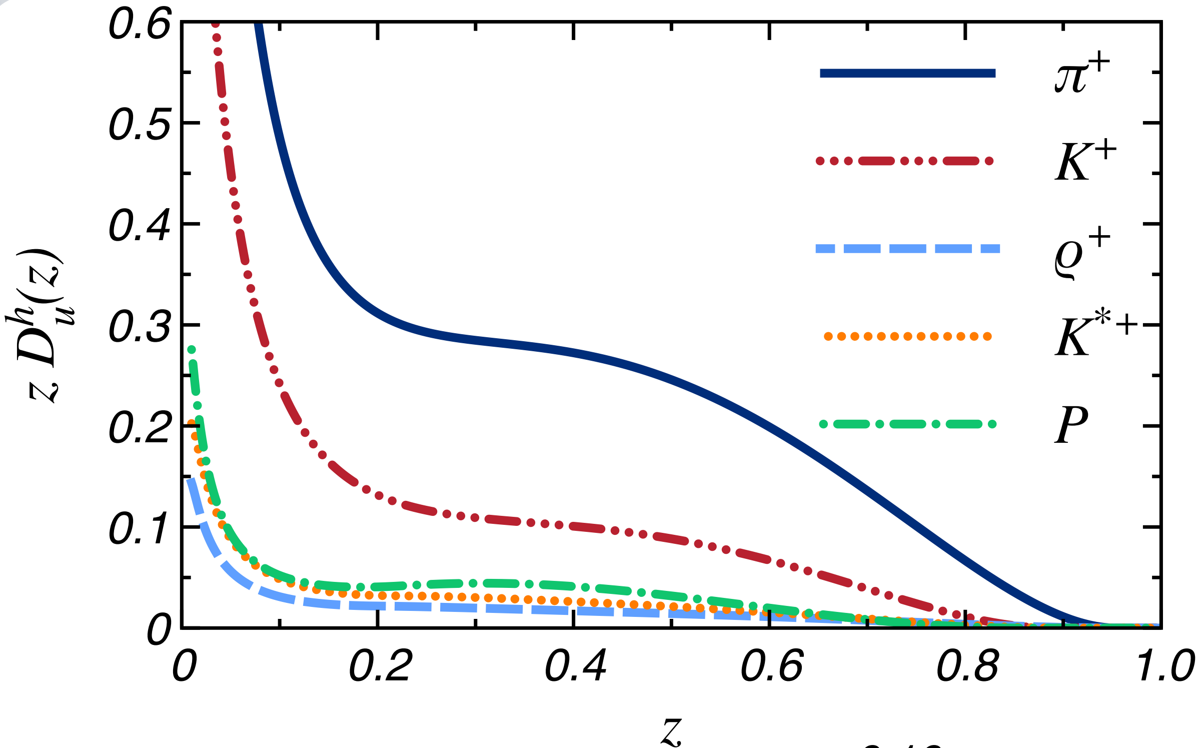
Favored

Unfavored



Results: Fragmentations to All Hadrons

$$Q^2 = 4\ \mathrm{GeV}^2$$



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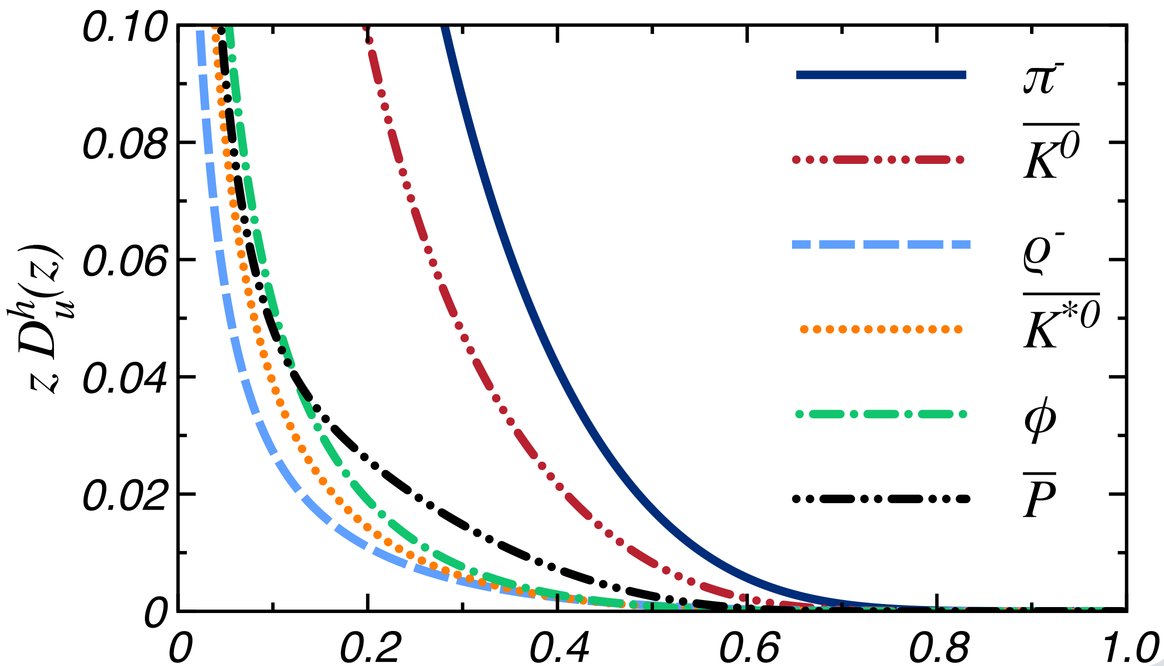
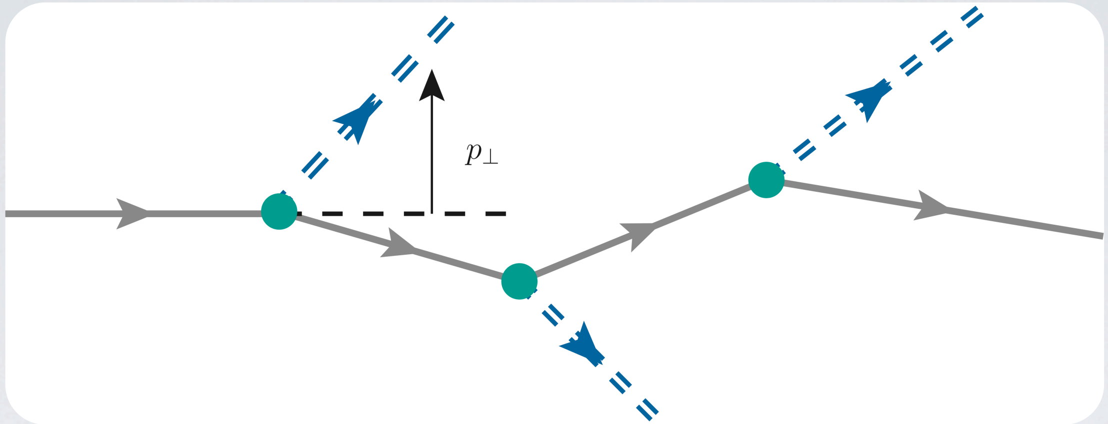


INCLUDING THE TRANSVERSE MOMENTUM

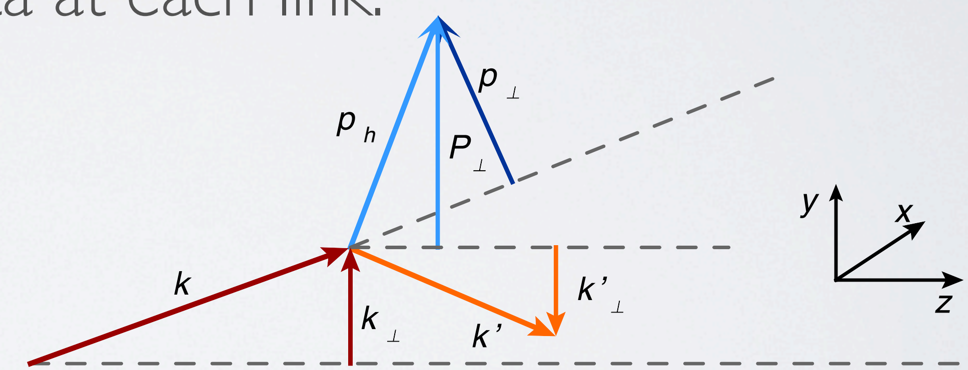


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

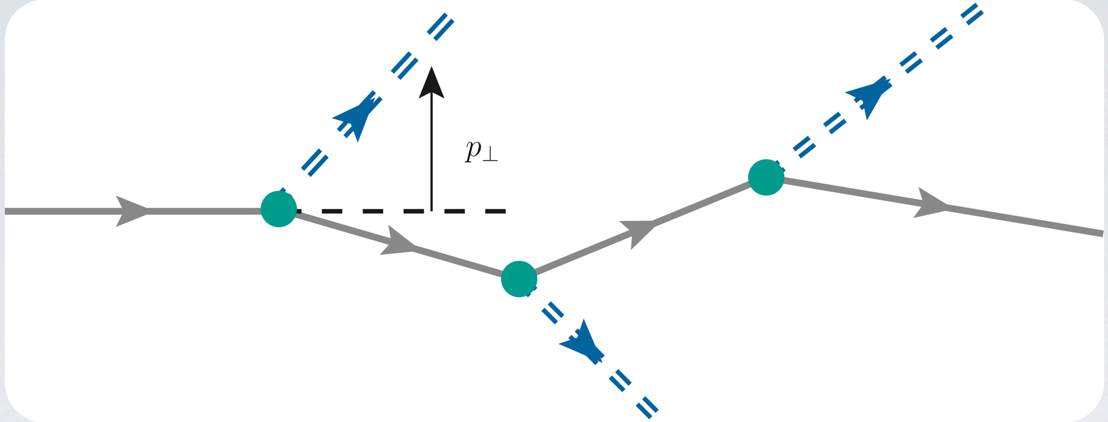
$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z \; \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

INCLUDING THE TRANSVERSE MOMENTUM



- TMD splittings: $d(z, p_\perp^2)$

Approximate $\mathcal{O}(k^2/Q^2)$

- Conserve transverse momenta at each link.

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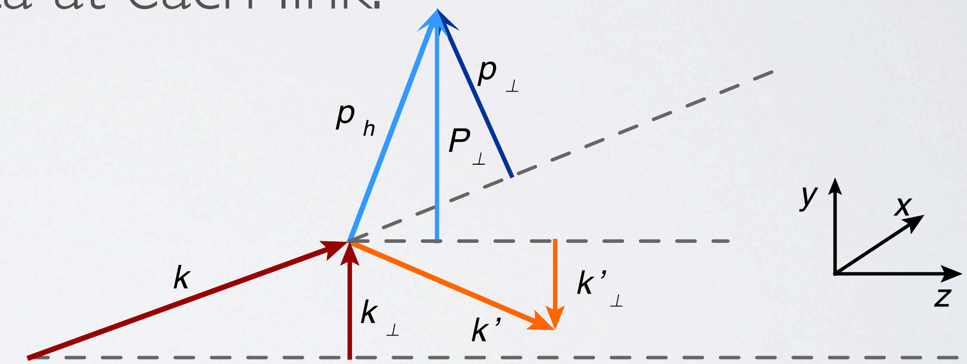

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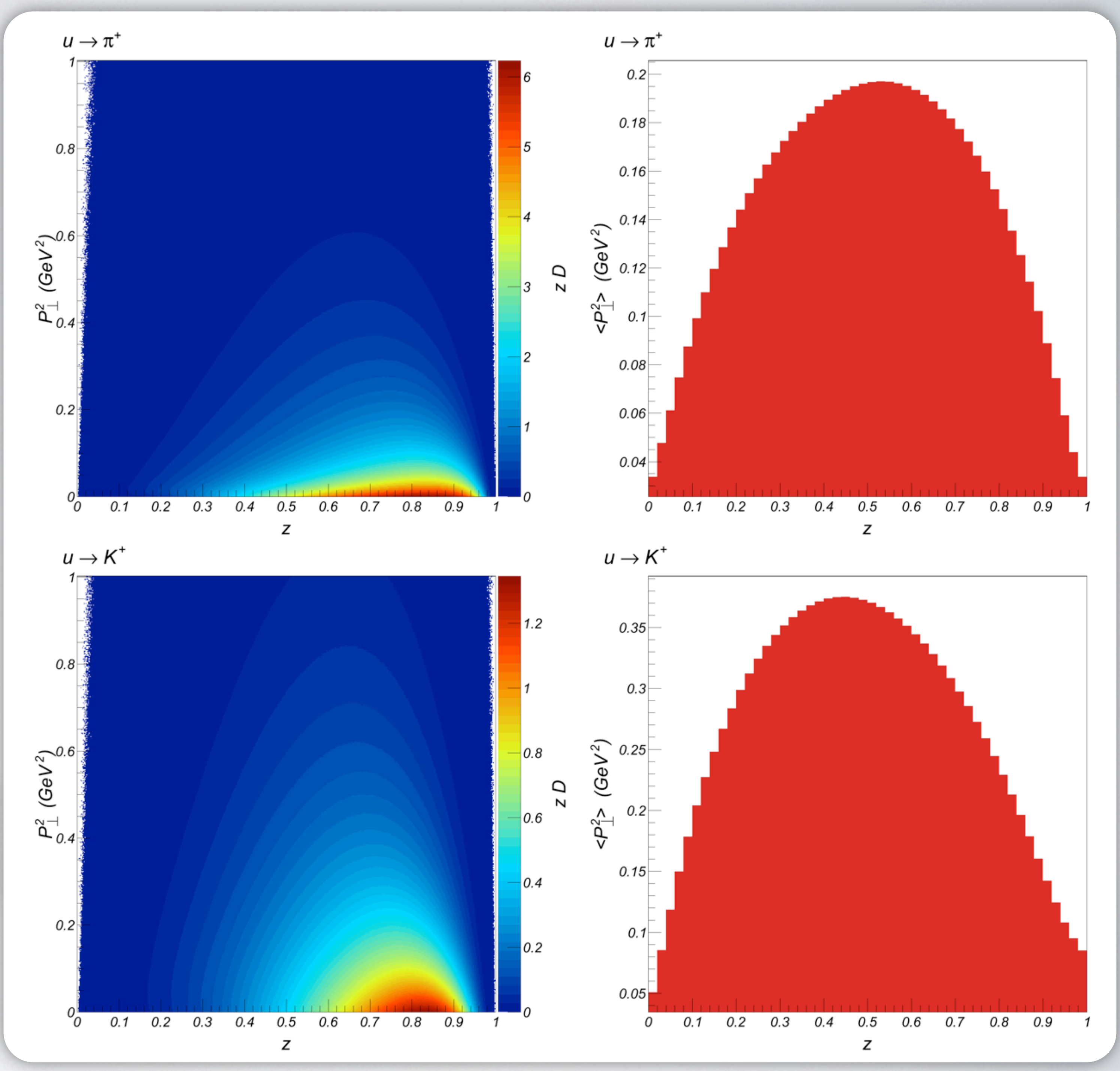
TMD SPLITTING FUNCTIONS

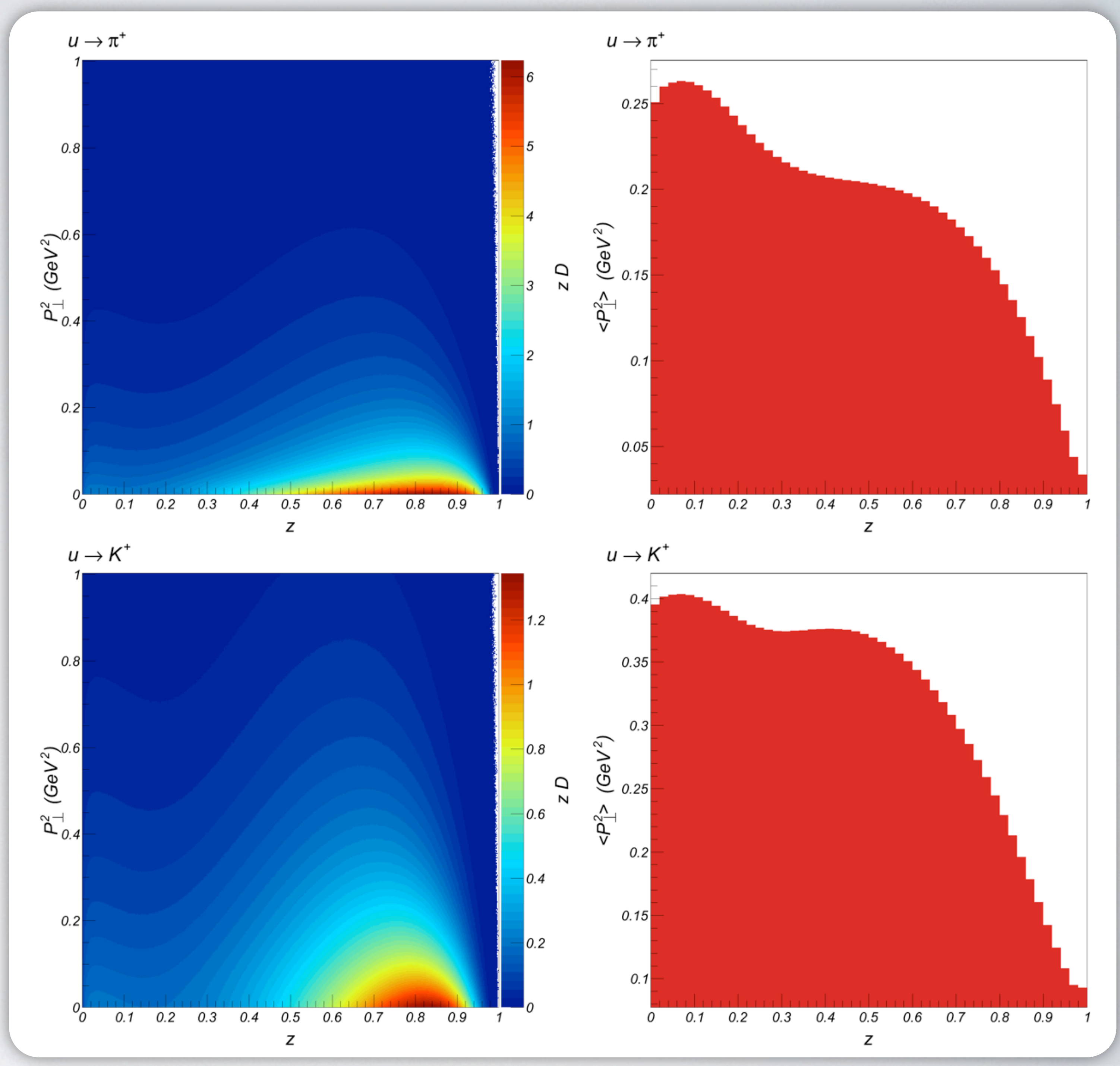
- TMD splittings from the NJL model
- Use dipole cutoff function with LB regularizations

$$\langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$$



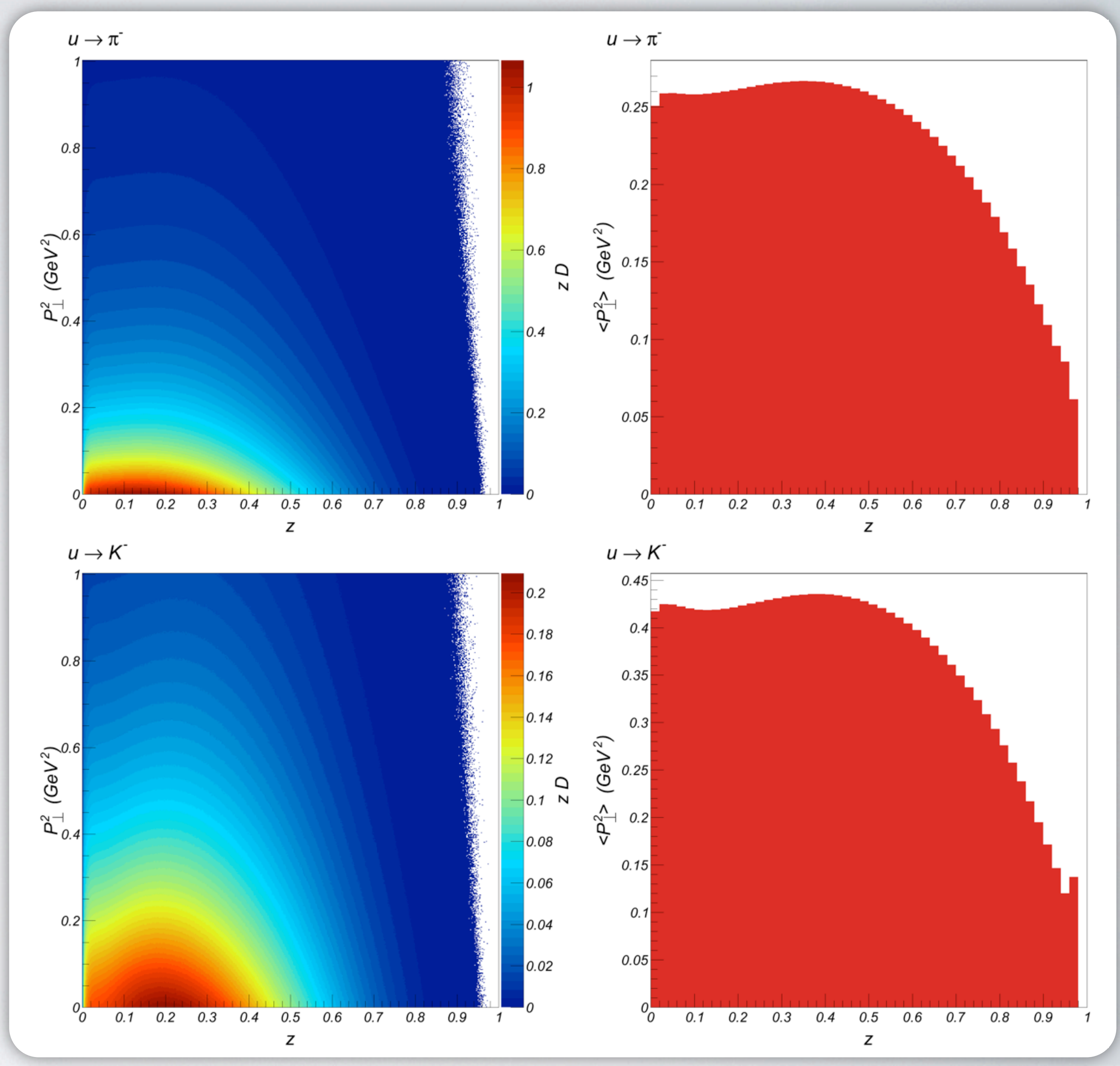
TMD FRAGMENTATION FUNCTIONS

- FAVORED

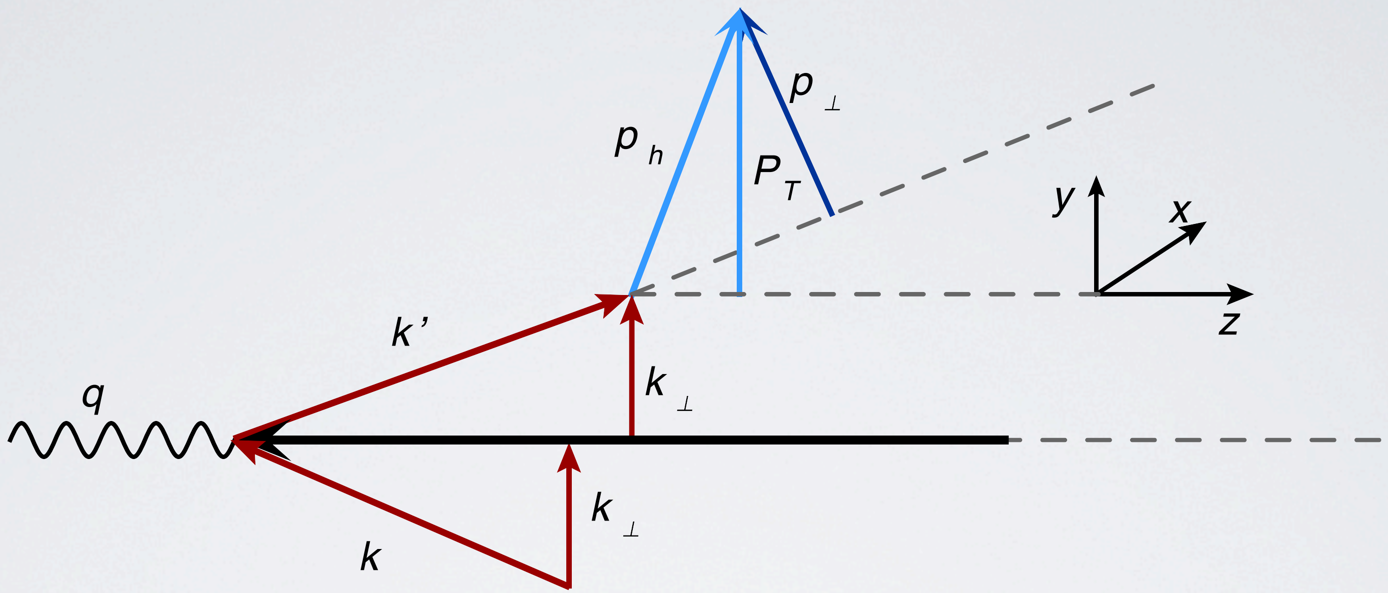


TMD FRAGMENTATION FUNCTIONS

- UNFAVORED



THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS

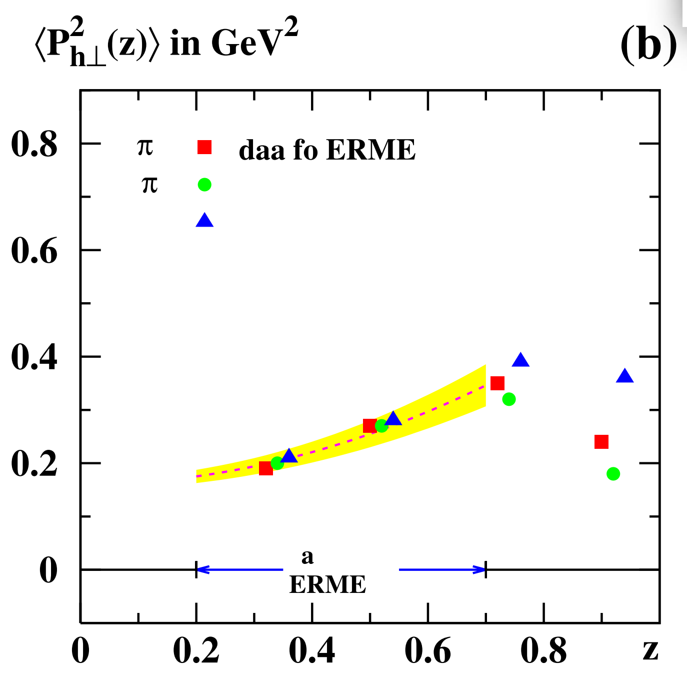


- Use TMD quark distribution functions calculated in the NJL model (see Ian Cloet's talk)
- Transfer of the transverse momentum: $\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k}_\perp$
- Evaluate $\langle P_T^2 \rangle$ using MC simulations to calculate the number densities

AVERAGE TRANSVERSE MOMENTA

$$\langle k_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{k}_\perp \; k_\perp^2 f(x, k_\perp^2)}{\int d^2\mathbf{k}_\perp \; f(x, k_\perp^2)} \qquad \langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$$

P. Schweitzer et al., Phys.Rev. D81, 094019 (2010).



Using Gaussian Ansatz and:

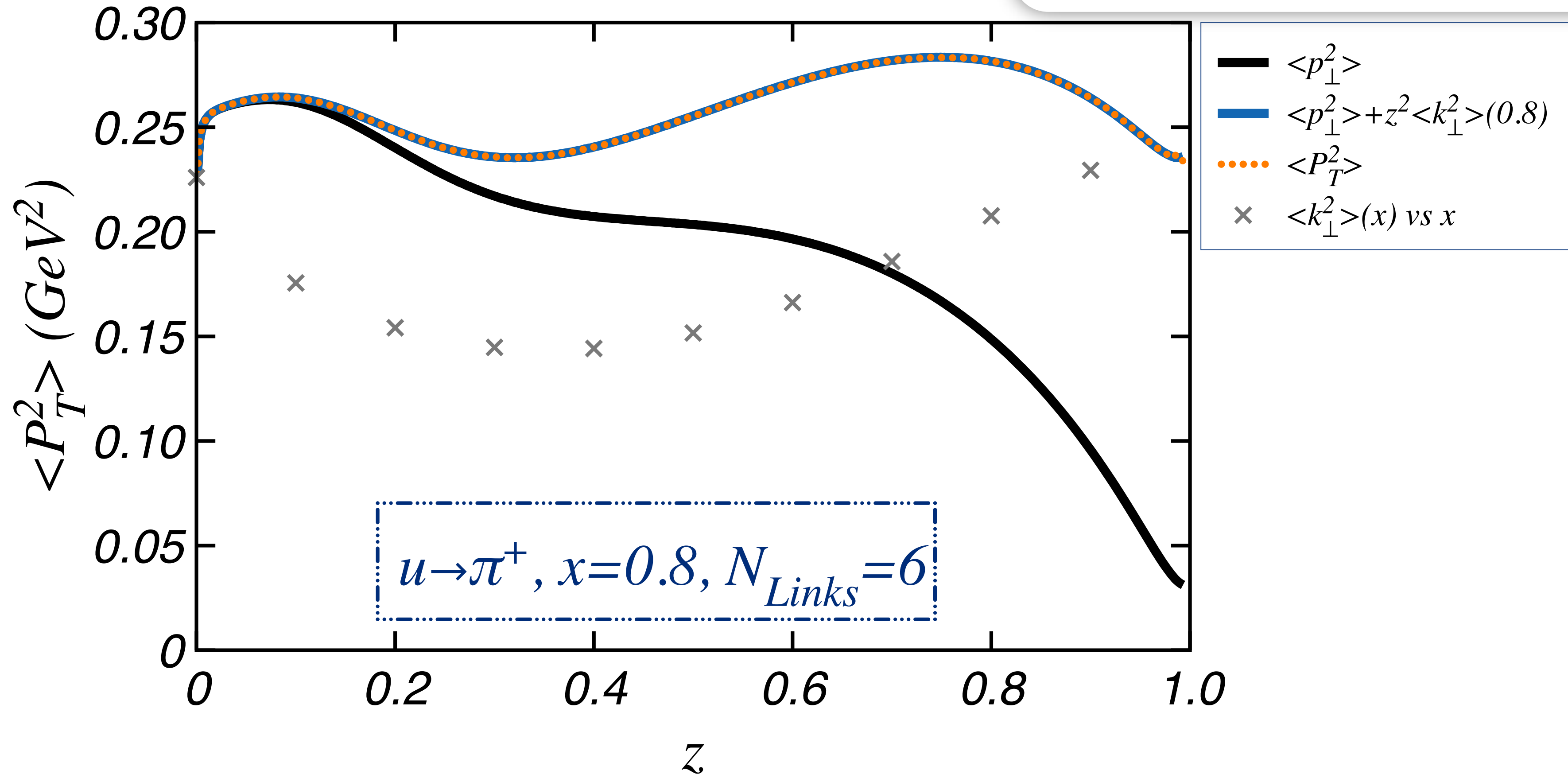
$$\langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_\perp \rangle$$

$$\langle k_\perp^2 \rangle = (0.38 \pm 0.06)\ \mathrm{GeV}^2$$

$$\langle P_\perp^2 \rangle = (0.16 \pm 0.01)\ \mathrm{GeV}^2$$

AVERAGE TRANSVERSE MOMENTA

$$\langle P_T^2 \rangle \equiv \frac{\int d^2\mathbf{P_T}\; P_T^2 \tilde{D}(z, P_T^2)}{\int d^2\mathbf{P_T}\; \tilde{D}(z, P_T^2)}$$



Input: $\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k}_\perp$

Output: $\langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_\perp \rangle$

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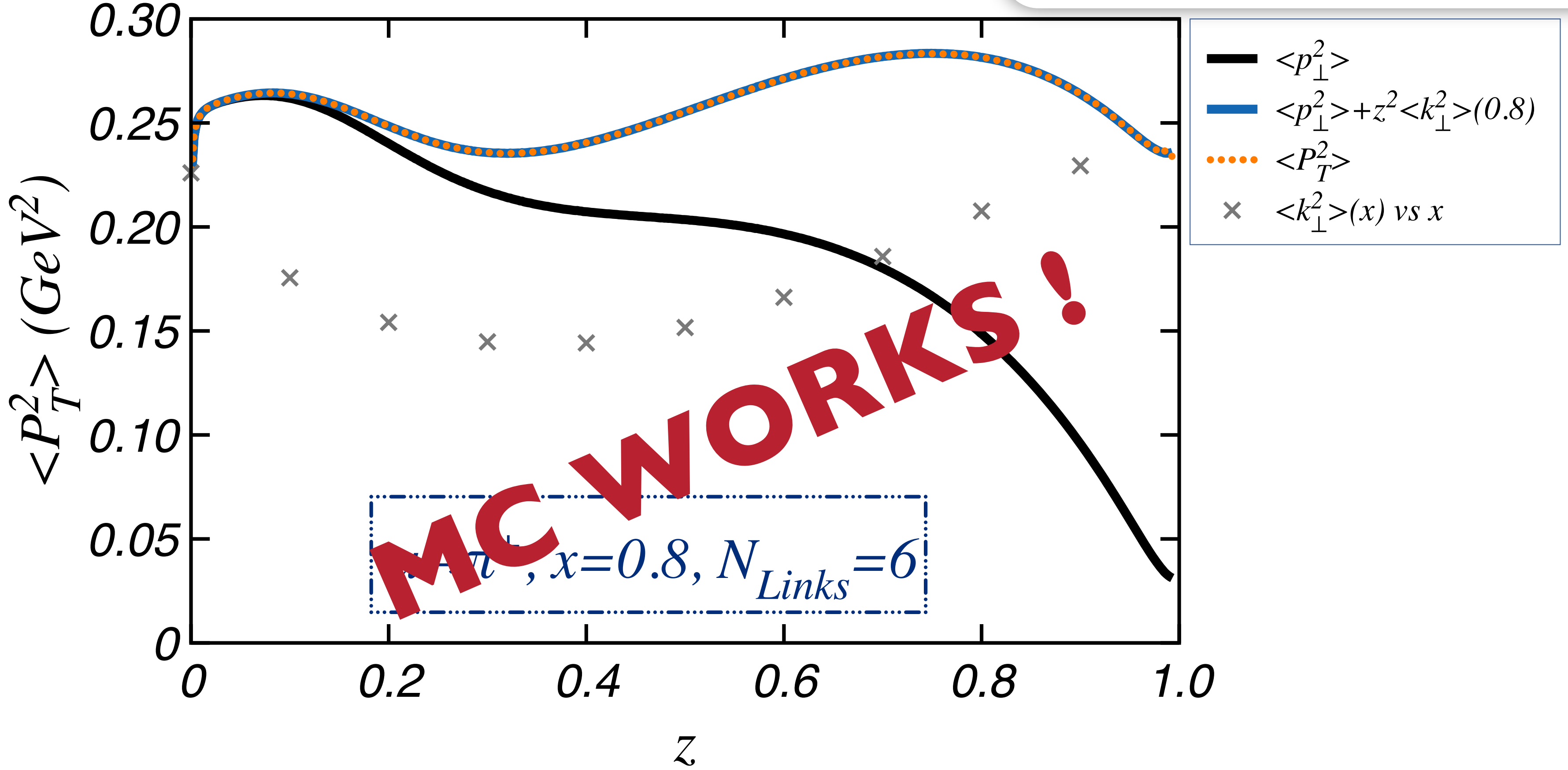

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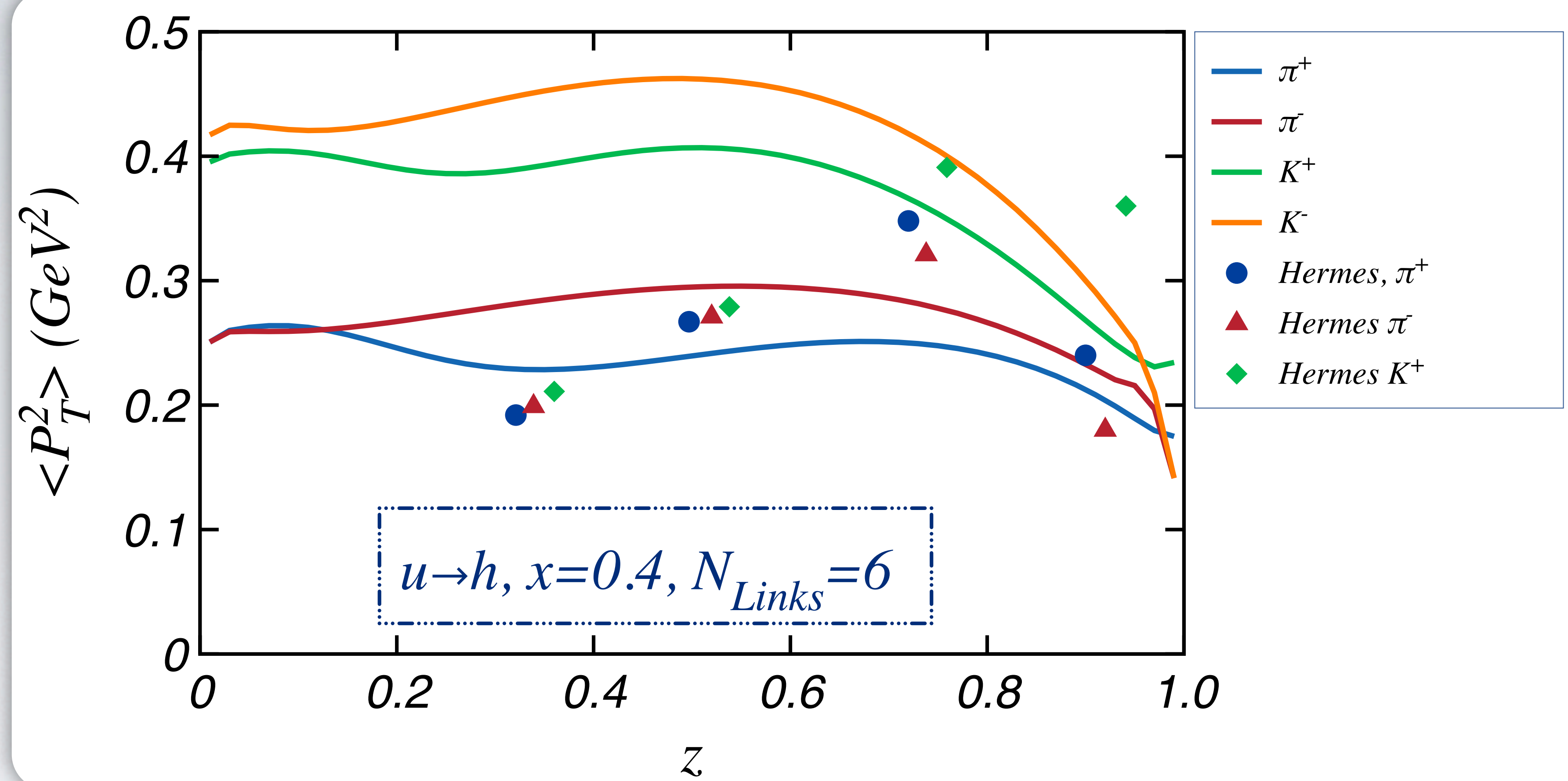
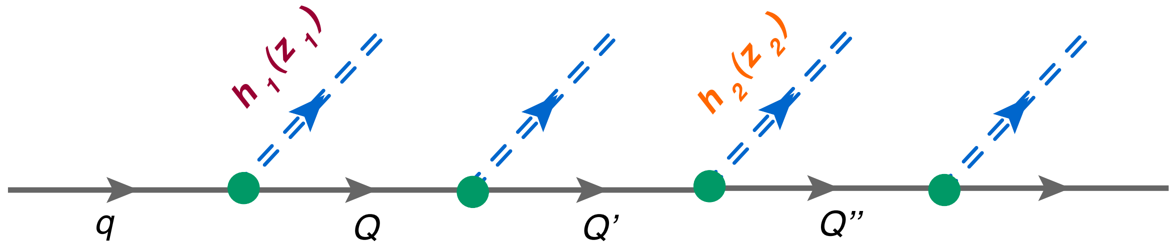
NAIVE COMPARISON WITH EXPERIMENT

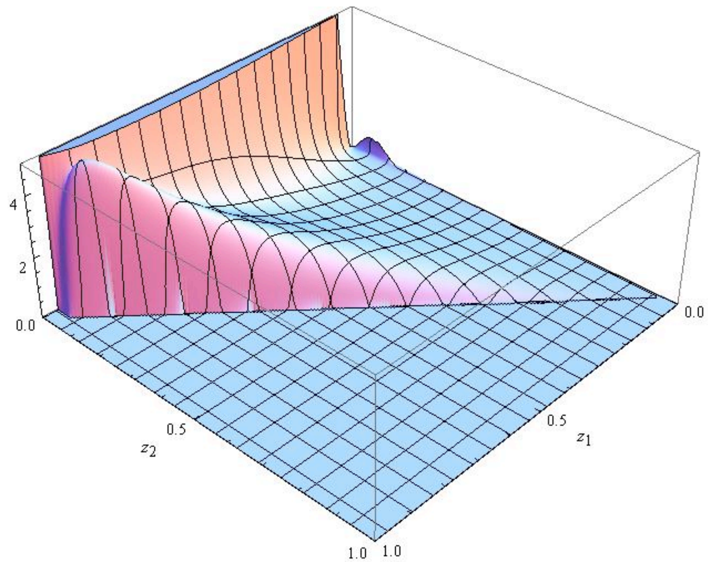


A. Airapetian et al. (HERMES Collaboration), Phys.Lett. B684, 114 (2010).

D target, Integration over Q^2 and x .

DIHADRON FRAGMENTATION FUNCTIONS





$$D_q^{h_1,h_2}(z_1,z_2) = \hat{d}_q^{h_1}(z_1)\frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1} + \hat{d}_q^{h_2}(z_2)\frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2} + \sum_Q \int_{z_1+z_2}^{1} \frac{d\eta}{\eta^2}\hat{d}_q^Q(\eta) D_Q^{h_1,h_2}(\frac{z_1}{\eta},\frac{z_2}{\eta})$$

See Andrew Casey's Talk on Wednesday at 17:35!

SUMMARY

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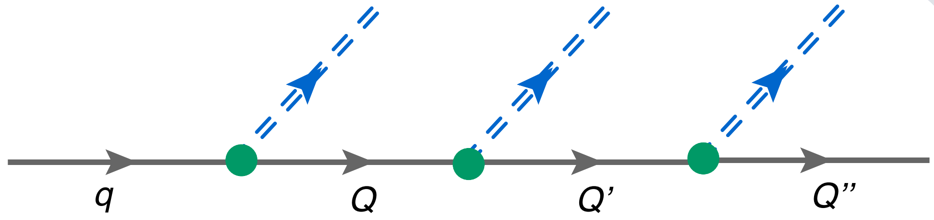
☑ 2009



Ito et al. Phys.Rev.D80:074008,2009

SUMMARY

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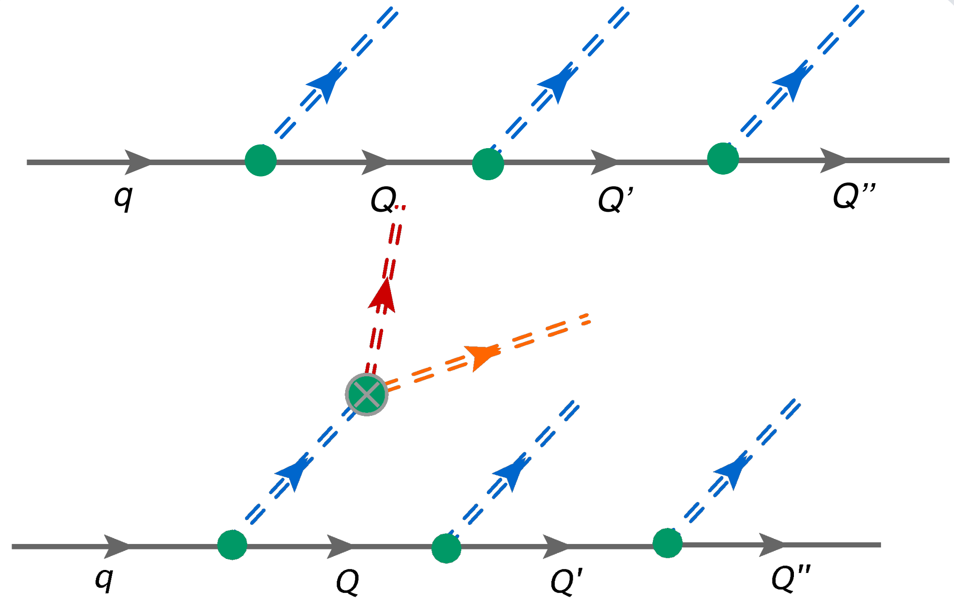

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Matevosyan et al.
Phys.Rev.D83:074003, 2011

Matevosyan et al.
Phys.Rev.D83:114010, 2011

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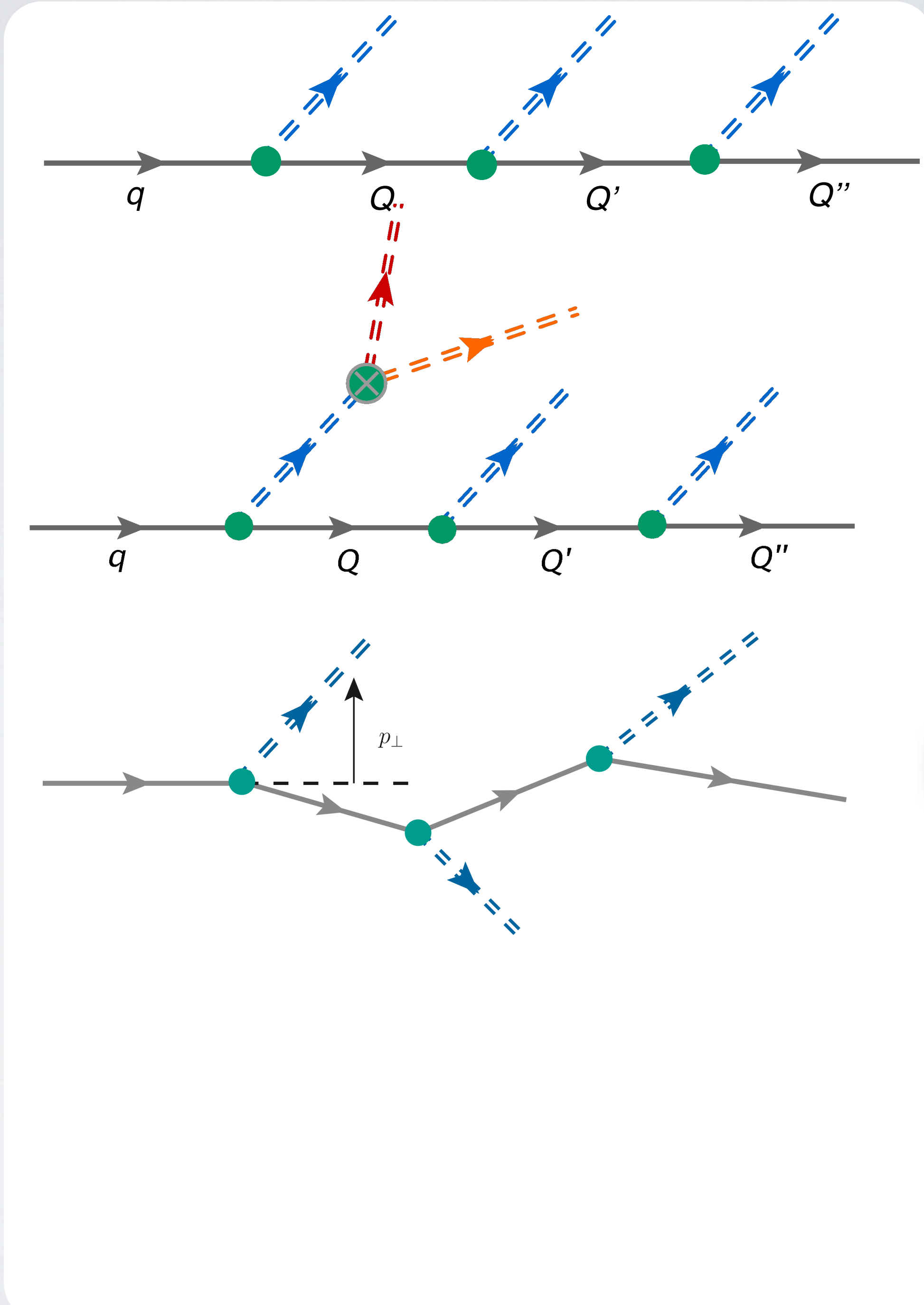
Coming Soon !



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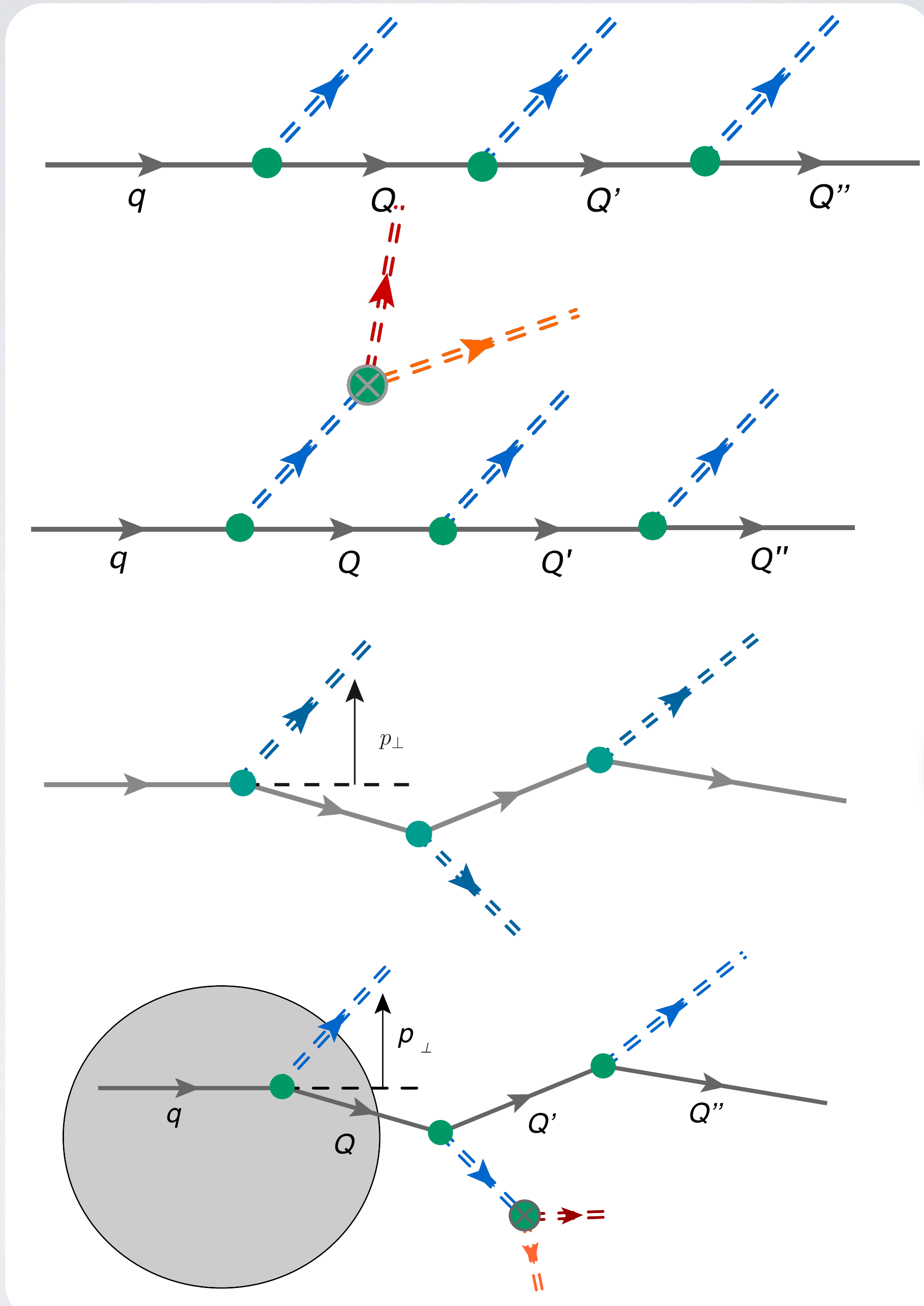
☑ 2011

Coming Soon !

☐ 2011-
2012




Cheers!

SETTING THE MODEL SCALE

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$$\alpha_s(M_z^2) = 0.118$$

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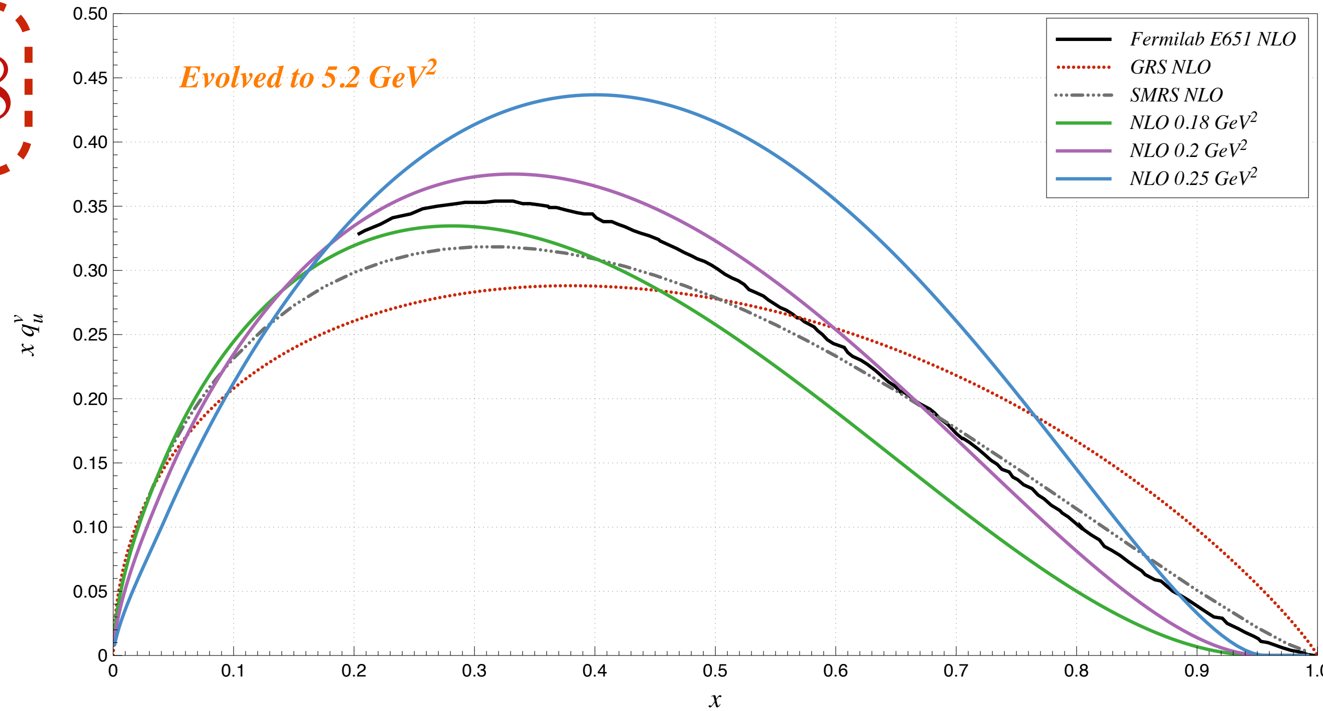
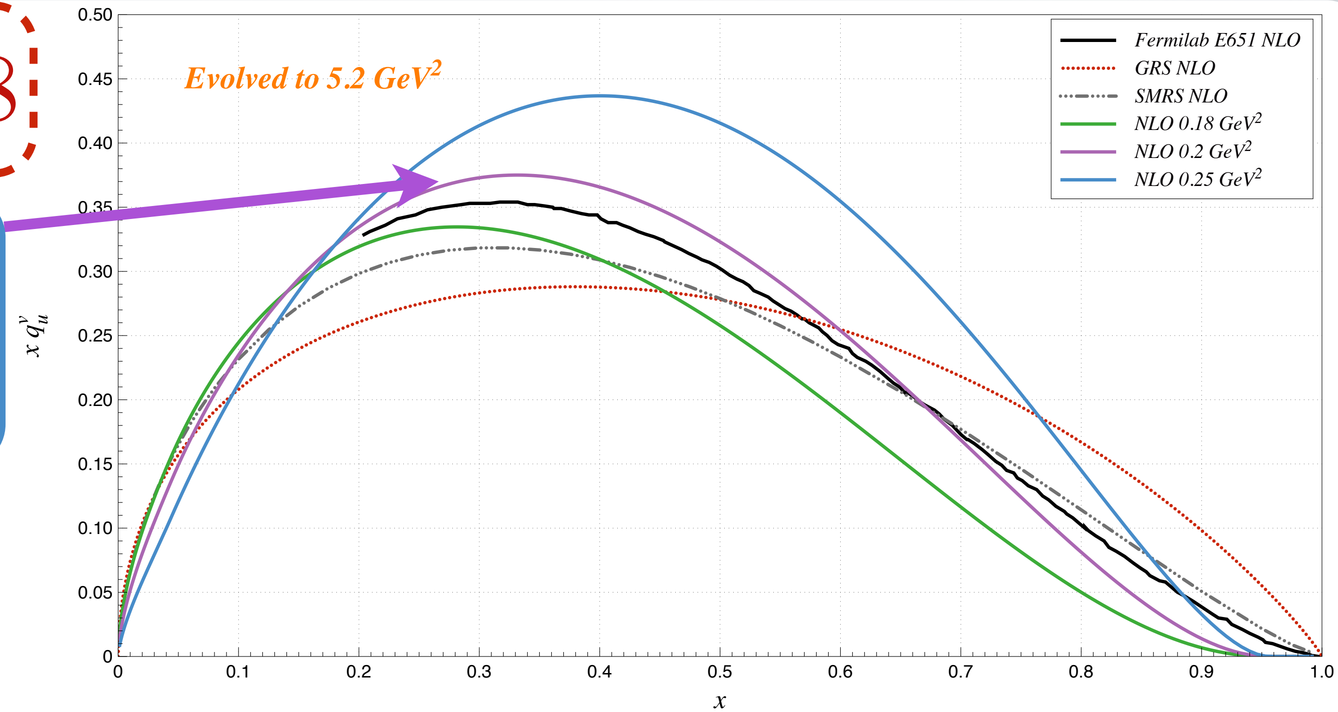


SETTING THE MODEL SCALE

$$\alpha_s(M_z^2) = 0.118$$

$$Q^2_{0\,NLO} = 0.2\ GeV^2$$

$$\alpha_s^{NLO}(Q_0^2) = 1.67$$



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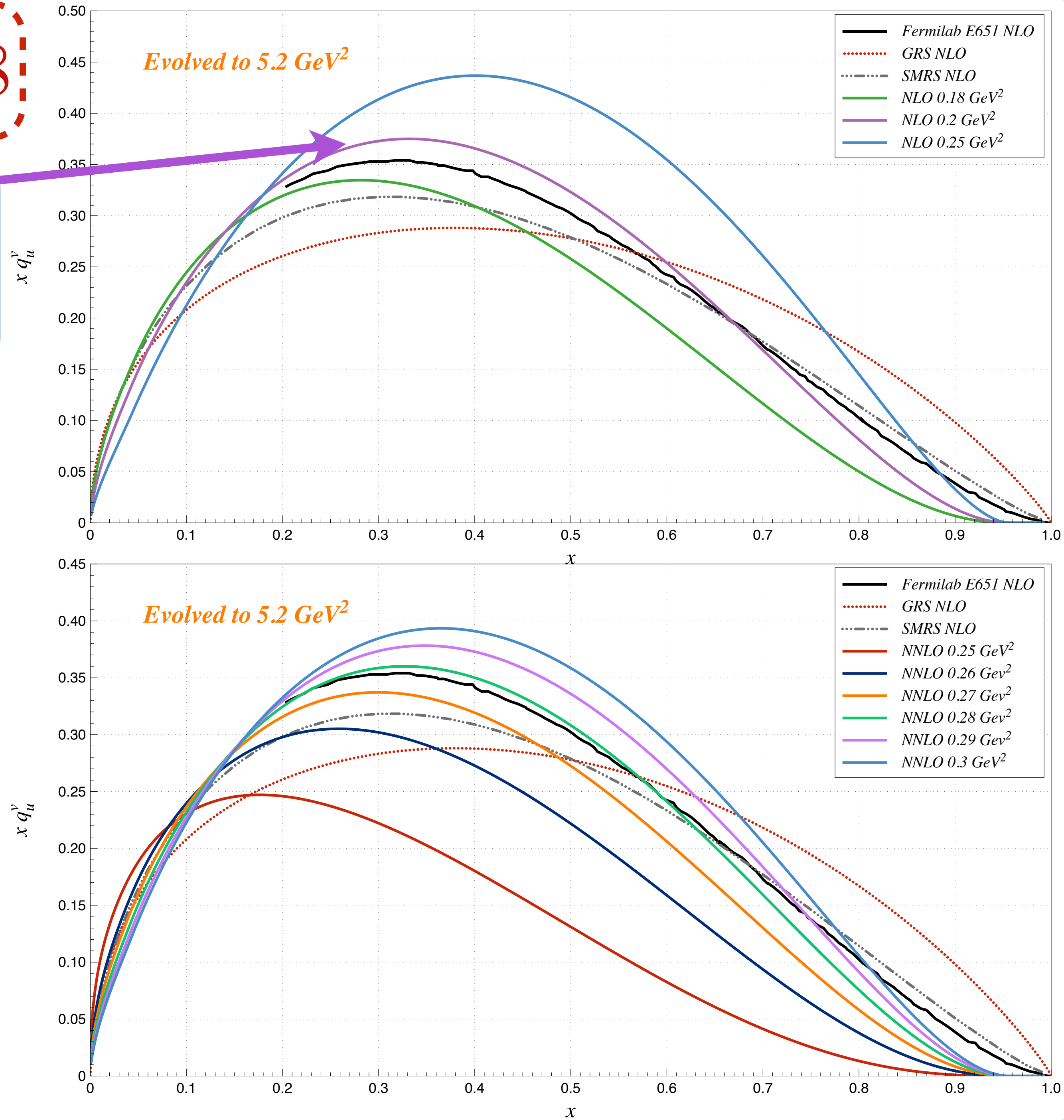

SETTING THE MODEL SCALE

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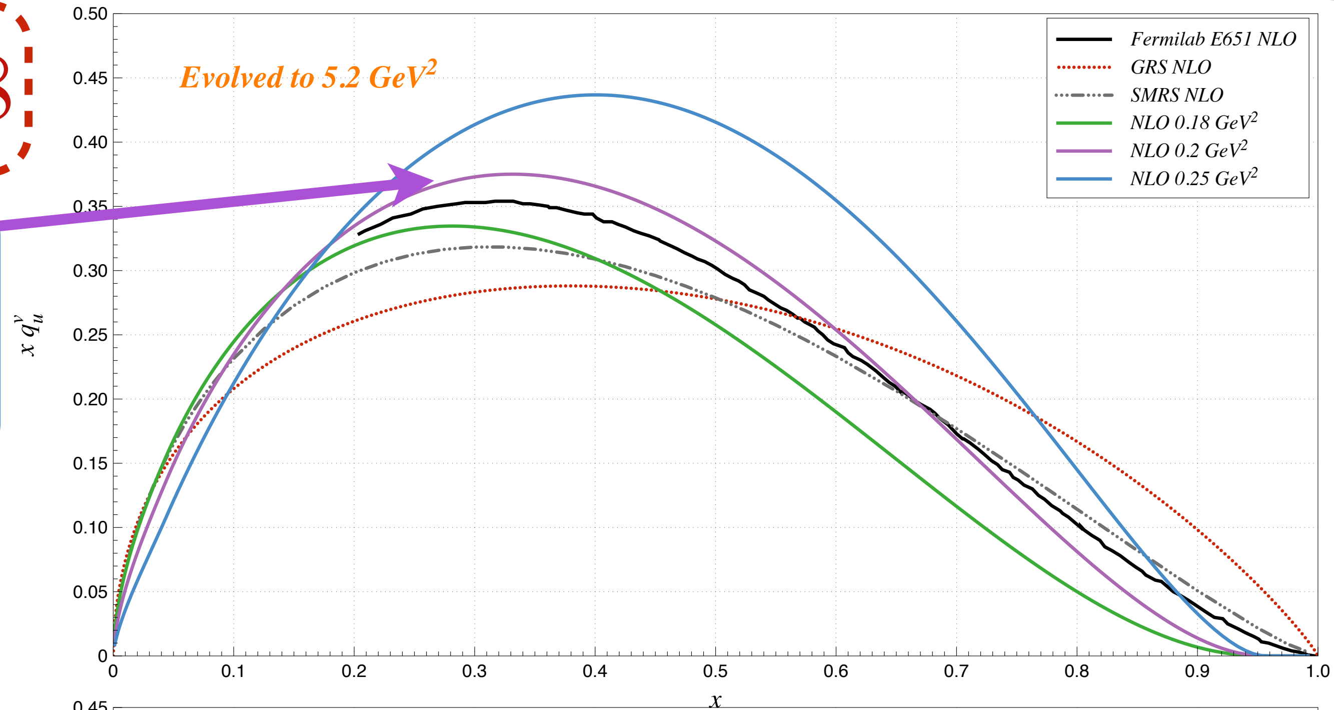
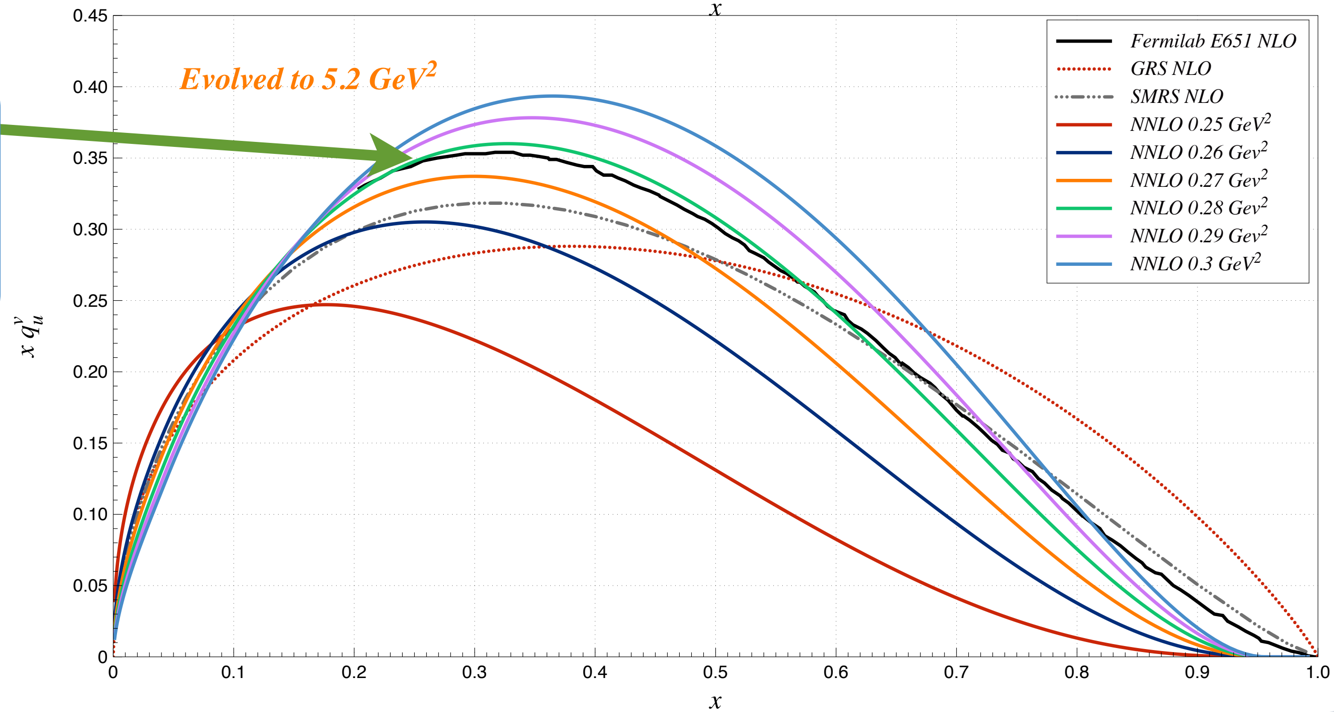
$$Q^2_{0NLO} = 0.2\ GeV^2$$

$$\alpha_s^{NLO}(Q_0^2) = 1.67$$



$$Q^2_{0NNLO} = 0.28\ GeV^2$$

$$\alpha_s^{NNLO}(Q_0^2) = 1.59$$



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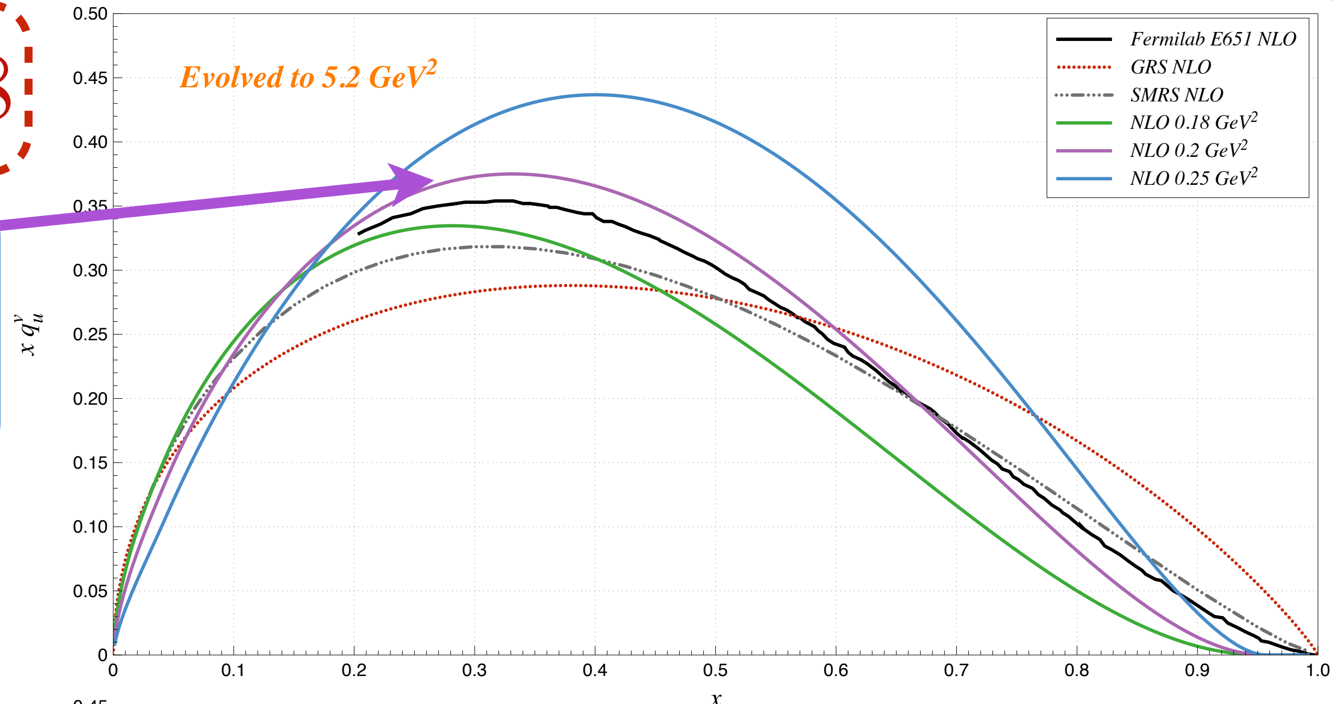

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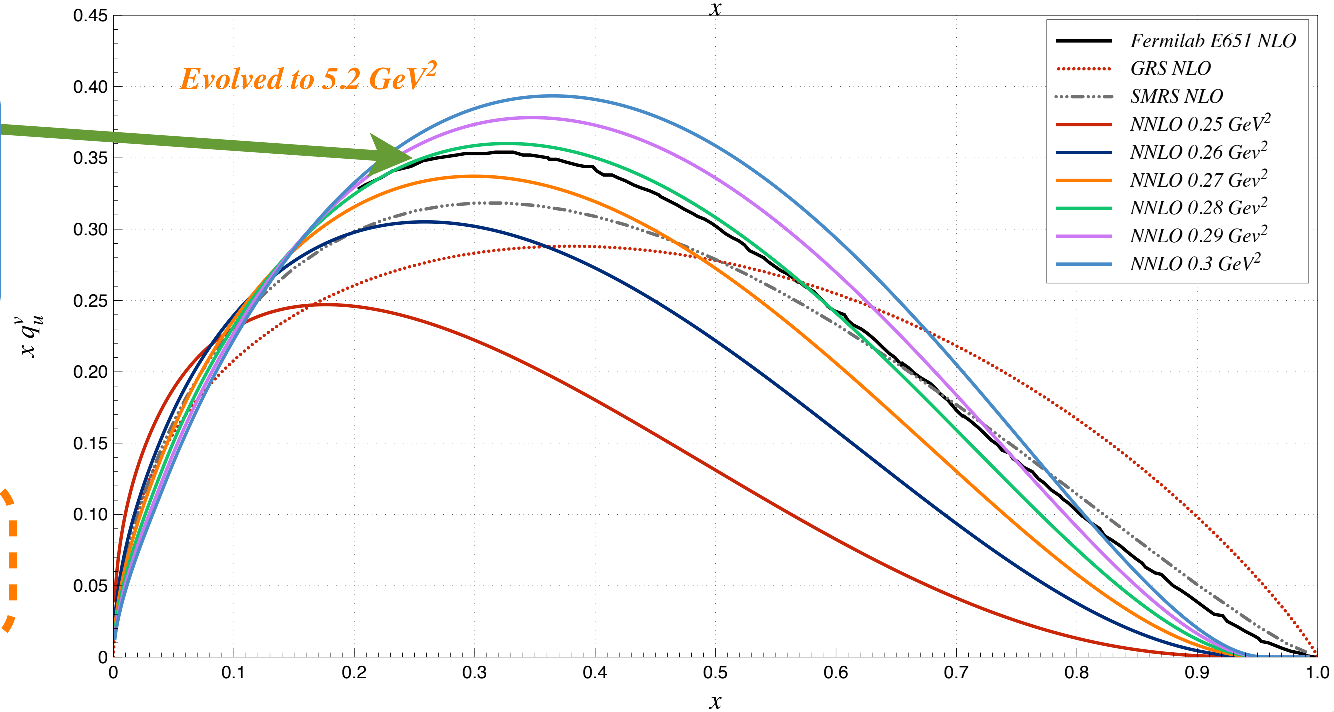

$$\alpha_s(Q_0^2)/2\pi \approx 0.25$$